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DESIGN AND IMPLEMENTATION OF A SIMULATOR FOR 3D FLEXIBLE-LINK SERIAL ROBOTS

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ABSTRACT

In this paper, an effective method in dynamic modeling of spatial flexible-link robots under large displacements and small deformations is discussed and a generic Matlab™ software simulator based on it is presented and validated. The adopted method is based on an Equivalent Rigid Link System (ERLS) that enables to decouple the kinematic equations of the ERLS from the compatibility equations of the displacements at the joints allowing an easy and recursive procedure to build the robot dynamic matrices.

The simulator is suitable for dynamic modelling of generic 3D serial flexible-link robots. The Matlab™ software simulator is validated with respect to the Adams-Flex™ commercial software, which implements Floating Frame of Reference (FFR) formulation, one of the most used methods for dynamic modeling of multibody flexible-link mechanisms with large displacements and small deformations.

Keywords: flexible-link mechanism, multibody systems, Equivalent Rigid Link System (ERLS).

INTRODUCTION

Dynamic modeling and control of flexible-link robotic systems is an important field of scientific investigations. Indeed, nowadays, the research effort focuses on the improvement of the dynamic performances of the robotic systems by increasing the velocity and lightening the structure while maintaining a high degree of accuracy. Thus, complex and effective dynamic models have to be used since both inertial and elastic effects have to be taken into account being hypothesis of rigid links no longer valid.

For multibody rigid flexible-link robotic systems, many dynamic models and formulations have been proposed in literature. The research covered the study of single flexible-link mechanisms, then planar and finally spatial flexible-mechanisms, and this topic is still an open field of investigation [1, 2, 13 and 16].

The classical approach used in multibody dynamics deals with mechanisms featuring large displacements and small deformations. Two main techniques have been adopted in literature [2, 4, 6, 7, 9 and 10]: the finite element method (nodal approach) and the assumed mode method (modal approach). Rigid body and elastic motion coupling effects have been considered in several works and approaches, firstly by considering only the effect of the rigid body motion on the elastic deformation [6, 9] and then by considering also the effect of the elastic deformation on the rigid body motion [10]. The outcome of these works is the Floating Frame of Reference (FFR) formulation [11, 12]. In the FFR formulation a first set of coordinates expresses the location and orientation of a local reference attached to each link, and a second set describes the deformation of the body with respect to its coordinate system. With this description a system of coupled differential equations is obtained being no separation between the rigid body motion and the elastic deformation of the flexible body. A possible drawback of this approach is that the constraint conditions, i.e. the connections between different deformable bodies, are defined in the global coordinate system: the resulting constraint equations are coupled and do not have an immediate and easy formulation. Moreover, they are usually introduced into the dynamic equations by means of a set of nonlinear algebraic constraint equations, which depend both on the elastic deformations and on the reference rigid motion of the deformable bodies (e.g. through a vector of Lagrange multipliers).

The method here proposed is intended for accurate dynamic modeling of systems with large displacements and small elastic deformation, and it is based on an Equivalent Rigid Link System (ERLS) concept introduced in [14, 15]. This approach allows to decouple the kinematic equations of the ERLS from the compatibility equations of the displacements at the joints without neglecting the mutual influence between rigid body motion and vibration. One of the main advantages of the approach is that the standard robotics concepts of 3D kinematics can be adopted to formulate and solve the ERLS kinematics. The results can then be used and easily integrated in the equations of the dynamic model of the flexible multibody system. In literature, an ERLS based approach suitable only for the particular case of planar mechanisms with revolute joints has been presented and used in [3, 5].Recently, the ERLS concept has been applied for modeling of 3D flexible mechanisms [17].

In this paper, the next sections cover the description of the kinematics and dynamics of the ERLS and of the flexible-link robotic system. Then, the generic Matlab™ software simulator is described. Finally the comparison between the FFR and ERLS results is presented.

THE ERLS FORMULATION

Kinematics

In the formulation here presented the elastic displacements are defined with respect to an Equivalent Rigid Link Mechanism (ERLS). Each link is subdivided into spatial beam finite elements modeled with the Euler-Bernoulli theory. If a fixed global reference frame {X, Y, Z} is defined, calling $\mathbf{u_i}$ and $\mathbf{r_i}$ the vector of the nodal elastic displacements of the i-th finite element and the vector of the nodal position and orientation for the i-th element of the ERLS respectively, the absolute nodal position and orientation of i-th finite element $\mathbf{b_i}$ with respect to the global reference frame is:

$$b_i = r_i + u_i \tag{1}$$

Let $\mathbf{w_i}$ and $\mathbf{v_i}$ be the position vector of the generic point of the i-th element of the ERLS and its elastic displacement respectively, the absolute position $\mathbf{p_i}$ of the generic point inside the i-th finite element is given by:

$$p_i = w_i + v_i \tag{2}$$

For each finite element, a local coordinate system {x_i, y_i, z_i}, which follows the ERLS motion, can be defined. A local reference frame can be expressed with respect to global one by exploiting the ERLS, a rigid mechanism, thus by means of a set of generalized coordinates q, the m-rigid degrees of mobility of the mechanism. So, the Denavit-Hartenberg (DH) notation can be adopted to describe the kinematics of the ERLS. The nodal position and orientation vector $\mathbf{r_i}$ for the i-th element can then be expressed with respect to a suitable local frame. All the $\mathbf{r_i}$**'s** can then be gathered into a unique vector $\mathbf{r}$, representing the position and orientation of the whole ERLS. The variation $\mathbf{dr}$, the velocity $\dot{\mathbf{r}}$ and acceleration $\ddot{\mathbf{r}}$ of the vector $\mathbf{r}$ can be expressed, through the Jacobian matrix, as a function of the variation, velocity and acceleration of the vector of the generalized coordinates.

In order to correctly account for the displacement interpolations inside the finite elements, a local to global transformation matrix $\mathbf{R_i}$ $(\mathbf{q})$ and a block-diagonal rotation matrix $\mathbf{T}^i_k(\mathbf{q})$ expressing the transformation from the frame k, in which are expressed the nodal elastic displacements of the i -th finite element $\mathbf{u}^k_i$, to the local reference frame, are defined. In order to apply the virtual work principle, the virtual displacements should be used. For the first term on the right hand side of the Eq. (2) expressed by means of virtual displacements, the interpolation function matrix $\mathbf{N_i}$ (x_i, y_i, z_i) can be used to interpolate infinitesimal rigid-body displacements if the proper reference frames are employed, while for the second term both virtual nodal elastic displacements $\delta\mathbf{u}^k_i$ and virtual displacements $\delta\mathbf{q}$ of the generalized coordinates have to be considered. The expression for the virtual displacements in the fixed reference frame becomes:

$$\begin{aligned} \delta p_i &= R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)\delta r_i^k \\ &+\delta R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)u_i^k + R_i(q)N_i(x_i,y_i,z_i)\delta T_k^i(q)u_i^k \\ &+R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)\delta u_i^k \end{aligned} \tag{3}$$

The nodal displacement $\mathbf{u}^k_i$ (the elastic displacement of the node) is assumed to be small with respect to the rigid body displacement of the ERLS.

By differentiating twice, the expression of the acceleration of a generic point inside the i-th finite element can be computed:

$$\begin{aligned} \ddot{p}_i &= R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)\ddot{r}_i^k + R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)\ddot{u}_i^k \\ &+2(\dot{R}_i(q)N_i(x_i,y_i,z_i)T_k^i(q) + R_i(q)N_i(x_i,y_i,z_i)\dot{T}_k^i(q))\dot{u}_i^k \\ &+(\ddot{R}_i(q)N_i(x_i,y_i,z_i)T_k^i(q) + 2\dot{R}_i(q)N_i(x_i,y_i,z_i)\dot{T}_k^i(q) \\ &+R_i(q)N_i(x_i,y_i,z_i)\ddot{T}_k^i(q))u_i^k \end{aligned} \tag{4}$$

where the term: $\ddot{\mathbf{r}}_\mathbf{i}^\mathbf{k}$ is the linear and angular acceleration of the i-th element of the ERLS expressed in the k-th reference frame.

If the kinematic entities of all the finite elements are grouped into a unique vector, taking into account Eq. (1), after differentiation holds:

$$db = du + dr = \begin{bmatrix} I & J \end{bmatrix} \begin{bmatrix} du \\ dq \end{bmatrix} \tag{5}$$

The coefficient matrix of the above Eq. (5) is not square; hence, a given configuration **db** of infinitesimal nodal displacements corresponds to more sets of increments $[du^T\ dq^T]$ of the generalized coordinates of the system. The easiest way to eliminate this redundancy is to force to zero a number of elements of **du** equal to the number of generalized coordinates of the ERLS. If **du** is partitioned into its independent part ($\mathbf{du}_{in}$) and into its zeroed part ($\mathbf{du}_0$), and if **J** is correspondingly partitioned, the elements forced to zero can be eliminated from Eq. (5):

$$db = \begin{bmatrix} I & J_{in} \\ 0 & J_0 \end{bmatrix} \begin{bmatrix} du \\ dq \end{bmatrix} \tag{6}$$

The square matrix of coefficient of Eq. (6) must be non-singular, which implies that the determinant of $\mathbf{J}_0$ must be different from zero and no ERLS singular configurations have to be encountered during the motion.

Dynamics

By applying the principle of virtual work, the dynamic equations can be obtained:

$$\sum_i \int_{v_i} \delta p_i^T \ddot{p}_i \rho_i dv + \sum_i \int_{v_i} \delta\varepsilon_i^T D_i \varepsilon_i dv = \sum_i \int_{v_i} \delta p_i^T g \rho_i dv + (\delta u^T + \delta r^T) f \tag{7}$$

$$\delta W^{inertia} + \delta W^{elastic} = -\delta W^{external}$$

where $\mathbf{D}_i$, $\boldsymbol{\epsilon}_i$ and $\boldsymbol{\rho}_i$ are respectively the stress-strain matrix, the strain vector and the mass density for the i-th element, **g** is the gravity acceleration vector, and **f** is the vector of the concentrated external forces and torques. The total virtual work is split into the integrals over element volumes $\mathbf{v}_i$ and in the virtual work due to **f**; **δu** and **δr** refer to all nodes of the model.

Compatibility Equations

The formulation of the compatibility equations, i.e. the constraints due to the kinematic pairs, has to take into account the frame in which the nodal elastic displacements $\mathbf{u}^k_i$ are expressed.

Two consecutive links have different local frames, which are fixed according to the DH notation. In order to impose the compatibility conditions, the elastic deformations of last node of the i-th link and those of the first node of the (i+1)-th link must be expressed in the same local frame. Within a kinematic chain, for all the beam elements of a link except the last (i.e. the one connected to the following link), $\mathbf{T}^i_{k,i}$ is the blocked-diagonal identity matrix, because the suitable reference frame **k** for expressing the elastic displacements $\mathbf{u}^k_i$ of each node of the beam element coincides with the local link frame i. On the other hand, the elastic displacements of the last node of the i-th link have to be rotated into the local frame of the (i+1)-th link, so that the kinematic entities are defined with respect to the same reference system. If an open chain mechanism is considered, the elastic displacements of the last node of the final link are all expressed in the same correct local link frame. . The compatibility equations at the joints are written and included considering only the elastic displacements and are never used explicitly, since they are automatically taken into account when assembling the system matrices, thus avoiding the need to write a set of nonlinear algebraic constraints equations.

Local Nodal and Global Equilibrium

Nodal elastic virtual displacements $\boldsymbol{\delta}\mathbf{u}^k_i$ and virtual displacements of the ERLS $\boldsymbol{\delta}\mathbf{r}^k_i$ are completely independent.

A first set of equilibrium equations, i.e. The local nodal equilibrium equations can be obtained from Eq. (7) by considering: $\boldsymbol{\delta}\mathbf{r}_i = 0$; $\boldsymbol{\delta}\mathbf{R}_i = 0$; $\boldsymbol{\delta}\mathbf{T}_i = 0$; $\boldsymbol{\delta}\mathbf{u}_i = 0$; so Eq. (3) becomes:

$$\delta p_i = R_i(q) N_i(x_i, y_i, z_i) T_k^i(q) \delta u_i^k \tag{8}$$

By considering Eq.s (4, 7, 8), the following holds:

$$\begin{aligned}
&\sum_i \int_{vi} \left[\delta u_i^{kT} T_{k,i}^{i\,T} N_i^T R_i^T \right] \Big[R_i N_i T_{k,i}^i \ddot{r}_i^k + R_i N_i T_{k,i}^i \ddot{u}_i^k \\
&+ 2(\dot{R}_i N_i T_{k,i}^i + R_i N_i \dot{T}_{k,i}^i) \dot{u}_i + (\ddot{R}_i N_i T_k^i + 2\dot{R}_i N_i \dot{T}_k^i + R_i N_i \ddot{T}_k^i) u_i^k \Big] \rho_i dv \\
&+ \sum_i \int_{vi} \left(\delta u_i^{kT} T_{k,i}^{i\,T} B_i^T \right) D_i B_i T_{k,i}^i u_i^k dv \\
&= \sum_i \int_{vi} \left(\delta u_i^{kT} T_{k,i}^{i\,T} N_i^T R_i^T \right) g \rho_i dv + (\delta u^T + \delta r^T) f
\end{aligned} \tag{9}$$

Where $\mathbf{T}^{\mathbf{i}}_{\mathbf{k,i}}$ is the block-diagonal matrix of the i-th element that expresses the relation between the appropriate k-th frame and the local i-th frame. The elements of the mass, Coriolis, gyroscopic damping, centrifugal stiffness and stiffness contributions can be obtained from the integrals appearing in Eq. (9).

Some terms contain the first and second order derivatives of the rotation matrices $\mathbf{R}_i$ and $\mathbf{T}^i_{k,i}$, which is a block diagonal rotation matrix, that need to be computed. The $\dot{\boldsymbol{R}}_i$ term can be easily expressed as:

$$\dot{R}_i = S(\omega_i) R_i \tag{10}$$

where $\mathbf{S}(\boldsymbol{\omega}_i)$ is the anti-symmetric skew matrix, function of the angular velocity $\boldsymbol{\omega}_i$. Hence, also the Coriolis terms can be computed as functions of **q** and $\dot{\boldsymbol{q}}$, because the angular velocity $\boldsymbol{\omega}_i$ depends on **q** and $\dot{\boldsymbol{q}}$. In a similar manner, the second order derivatives and, thus, the centrifugal stiffness terms, can be expressed in an efficient and simple formulation.

Finally, for the local nodal equilibrium expressed in Eq. (9), results:

$$\begin{aligned}
&\sum_i \delta u_i^{kT} M_i (\ddot{r}_i + \ddot{u}_i) + 2\sum_i \delta u_i^{kT} (M_{G1i} + M_{G2i}) \dot{u}_i^k \\
&+ \sum_i \delta u_i^{kT} (M_{C1i} + 2M_{C2i} + M_{C3i}) u_i^k + \sum_i \delta u_i^{kT} K_i u_i^k \\
&= \sum_i \delta u_i^T f_{gi} + \delta u^T f
\end{aligned} \tag{11}$$

A second set of equilibrium equations, i.e. global equilibrium, can be obtained by considering: $\delta \mathbf{q_j} \neq 0$, j = 1... n; $\delta \mathbf{u_i} = 0$; So, Eq. (3) becomes:

$$\delta p_i = R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)\delta r_i^k$$
$$+\delta R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)u_i^k \qquad (12)$$
$$+R_i(q)N_i(x_i,y_i,z_i)\delta T_k^i(q)u_i^k$$

If δRi, δTi and δri terms are expressed as:

$$\delta R_i = \sum_j (\partial R_i / \partial q_i)\delta q_j = R_i'\delta q \neq 0$$
$$\delta T_i = \sum_j (\partial T_i / \partial q_i)\delta q_j = T_i'\delta q \neq 0 \qquad (13)$$
$$\delta r_i = \sum_j (\partial r_i / \partial q_i)\delta q_j = J_i \delta q \neq 0$$

Eq. (12) results:

$$\delta p_i = R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)(J_i^k(q)\delta q)$$
$$+(R_i'(q)\delta q)N_i(x_i,y_i,z_i)T_k^i(q)u_i^k \qquad (14)$$
$$+R_i(q)N_i(x_i,y_i,z_i)(T_k^{i\prime}(q)\delta q)u_i^k$$

Now, if the orders of magnitude of the three terms in Eq. (14) are compared, it can be said that all the terms contain the **q** virtual displacements and matrices of the same order of magnitude.

The second and third term count the **u** vector that is small (i.e. we are in the small displacement condition). Thus, the second and third terms are here neglected being their order of magnitude lower with respect to the first term. Eq. (14) becomes:

$$\delta p_i = R_i(q)N_i(x_i,y_i,z_i)T_k^i(q)J_i^k(q)\delta q \qquad (15)$$

If Eq.s (4, 8, and 15) are considered, the following expression can be obtained:

$$\sum_i \int_{vi} \left[\delta q^T J_i^{kT} T_{k,i}^{i\;T} N_i^T R_i^T\right]\left[R_i N_i T_{k,i}^i \ddot{r}_i^k + R_i N_i T_{k,i}^i \ddot{u}_i^k\right.$$
$$\left.+2(\dot{R}_i N_i T_{k,i}^i + R_i N_i \dot{T}_{k,i}^i)\dot{u}_i + (\ddot{R}_i N_i T_k^i + 2\dot{R}_i N_i \dot{T}_k^i + R_i N_i \ddot{T}_k^i)u_i^k\right]\rho_i dv \qquad (16)$$
$$+\sum_i \int_{vi} \left(u_k^{i\;T} T_{k,i}^{i\;T} B_i^T\right) D_i B_i T_{k,i}^i u_i^k dv$$
$$= \sum_i \int_{vi} \left(\delta q^T J_i^{kT} T_{k,i}^{i\;T} N_i^T R_i^T\right) g\rho_i dv + \delta r^T f$$

The integrals that rise from the inertia virtual work term are the same previously evaluated. The $\delta \boldsymbol{T}_{k,i}^{i,T}$ terms in the elastic virtual work term in Eq. (16) can be transformed into an equivalent form by taking into account the Eq. (13):

$$\sum_i \int_{vi} u_i^T \delta T_{k,i}^{i\;T} B_i^T D_i B_i T_{k,i}^i u_i dv$$
$$= \sum_i \int_{vi} u_i^T \delta q^T T_{k,i}^{i'\,T} B_i^T D_i B_i T_{k,i}^i u_i dv \qquad (17)$$
$$= \sum_i u_i^T \delta q^T \left(\int_{vi} T_{k,i}^{i'\,T} B_i^T D_i B_i T_{k,i}^i dv\right) u_i = \sum_i u_i^T \delta q^T K_{1,i} u_i$$

Thus, for the global equilibrium results:

$$\delta q^T J^T \left[M(\ddot{r}+\ddot{u}) + 2(M_{G1}+M_{G2})\dot{u}\right.$$
$$\left.+(M_{C1}+2M_{C2}+M_{C3})u\right] + \sum_i u_i^T \delta q^T K_{1,i} u_i \qquad (18)$$
$$= \delta q^T J^T \left(f_g + f\right)$$

Now, by considering the equilibrium of the elastic forces with respect to all others in Eq. (11) and substituting in Eq. (18), this latter can be rewritten as:

$$\sum_i u_i^T \delta q^T K_{1,i} u_i - \sum_i \delta q^T J_i^T K_i u_i = 0 \qquad (19)$$

This equation shows that the first term can reasonably be neglected, because small displacement assumption ensures that $\boldsymbol{u^T \delta q^T k_{1,i} u}$ is negligible with respect to $\boldsymbol{\delta q^T J_i^T K_i u_i}$. The following system of differential equations, that contains local nodal and global equilibrium equations, is obtained by computing the sums for all the elements of the mechanism:

$$\delta u^T \left[M(\ddot{r}+\ddot{u}) + 2(M_{G1}+M_{G2})\dot{u}\right.$$
$$\left.+(M_{C1}+2M_{C2}+M_{C3})u\right] + \delta u^T K u \qquad (20)$$
$$= \delta u^T \left(f_g + f\right)$$

$$\delta q^T J^T \left[M(\ddot{r}+\ddot{u}) + 2(M_{G1}+M_{G2})\dot{u}\right.$$
$$\left.+(M_{C1}+2M_{C2}+M_{C3})u\right] + = \delta q^T J^T \left(f_g + f\right) \qquad (21)$$

The infinitesimal displacements of the ERLS can be expressed by means of the Jacobian matrix so that **δu's** and the **δq's** can be eliminated from Eq. (20) and Eq. (21). Hence, the following system of differential equations is obtained:

$$M(\ddot{r}+\ddot{u}) + 2(M_{G1}+M_{G2})\dot{u}$$
$$+(M_{C1}+2M_{C2}+M_{C3})u + Ku = \left(f_g + f\right) \qquad (22)$$

$$J^T M(\ddot{r}+\ddot{u}) + 2J^T(M_{G1}+M_{G2})\dot{u}$$
$$+J^T(M_{C1}+2M_{C2}+M_{C3})u = J^T\left(f_g + f\right) \qquad (23)$$

Where, again, Eq. (22) is a statement of nodal equilibrium, i.e. equivalent loads applied to each node must be in equilibrium, whereas Eq. (23) is a statement of overall equilibrium, i.e. all equivalent nodal loads applied to the linkage produces no work for a virtual displacement of the ERLS.

In a realistic situation dealing with flexible manipulator systems, damping is usually present and is here taken into account by using the Rayleigh model of damping. Thus, Eq. (22) and Eq. (23) become:

$$M(\ddot{r}+\ddot{u}) + 2(M_{G1}+M_{G2})\dot{u} + \alpha M\dot{u} + \beta K\dot{u}$$
$$+(M_{C1}+2M_{C2}+M_{C3})u + Ku = \left(f_g + f\right) \qquad (24)$$

$$J^T M(\ddot{r}+\ddot{u}) + 2J^T(M_{G1}+M_{G2})\dot{u} + \alpha J^T M u$$
$$+J^T(M_{C1}+2M_{C2}+M_{C3})u = J^T\left(f_g + f\right) \tag{25}$$

Dynamic equations, after the substitution of the second order differential kinematics equations of the ERLS, can be grouped and rearranged in matrix form after discarding the equations for the elastic degrees of freedom that have been zeroed:

$$\begin{bmatrix} M & MJ \\ J^T M & J^T MJ \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{q} \end{bmatrix} =$$
$$\begin{bmatrix} -2\left(M_{G1}+M_{G2}\right)-\alpha M-\beta K & -M\dot{J} & -\left(M_{C1}+2M_{C2}+M_{C3}\right)-K \\ J^T\left(-2\left(M_{G1}+M_{G2}\right)-\alpha M\right) & -J^T MJ & -J^T\left(M_{C1}+2M_{C2}+M_{C3}\right) \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{q} \\ q \end{bmatrix}$$
$$+\begin{bmatrix} M & I \\ J^T M & J^T \end{bmatrix}\begin{bmatrix} g \\ f \end{bmatrix} \tag{26}$$

In this way, the values of the accelerations can be computed at each step by solving the system Eq. (26), while the values of velocities and of displacements can be obtained by an appropriate integration scheme (e.g. the Runge-Kutta algorithm).

MATLAB^{TM} SOFTWARE SIMULATOR

According to the method mentioned in previous sections, a generic Matlab^{TM} software simulator has been developed which is suitable for modeling and dynamic simulating serial flexible-link 3D robots.

Since the ERLS formulation allows to exploit the DH notation and the main concepts of robotics kinematics, it allows to study the dynamics flexible-link robotic systems without differences with respect to the rigid ones. This can be considered an useful feature of the formulation and allows to justify the implementation of a generic software based on the ERLS.

The Matlab^{TM} simulator is structured in three main parts:

a) The first is related to the DH, geometrical and mechanical parameters definition.

In this part, the main concepts of robotics kinematics, e.g. Denavit-Hartenberg notation, have been exploited in order to give to the user the possibility to create a generic serial robot.

The starting page of the simulator user Interface is presented as a list of pre-analyzed robots which have been previously evaluated from the symbolic point of view. In this case, the user can decide which configuration of the robots wants to load. Robots provided by default have been chosen considering the benchmarks proposed by the literature of multibody dynamics and the most common serial spatial robots. They are as follows:

- Simple pendulum;
- Planar double pendulum;
- Spatial double pendulum;
- Anthropomorphic manipulator;
- Anthropomorphic robot with spherical wrist;
- Manipulator DLR with spherical wrist.

Alternatively, the user can place any other mechanisms to analyze with pressing the **New Configuration** button. The first menu of the simulator is shown in Figure 1.

Once one of the options is chosen by the user, a second interface is loaded; in this page the kinematic, geometrical and mechanical data essential for the definition of the robot are required .

To describe unambiguously the type of manipulator that is to be analyzed from the kinematic point of view, the simulator asks **DH** parameters in this page.

The additional parameters that have to be entered by the user are related to characteristics of each link of the robot. In particular, the length **L** [m], the width in **y** direction [m], the depth in the **z** direction [m], the density [kg /m^3], the module of elasticity **E** [N / m^2], the coefficient of Poisson **ν** and, finally, the number of elements for each link have to be defined.

Also, the user must enter other effective parameters on the mechanism performance, like the direction the force of gravity which can be in the direction of **y** axis or **z** axis, and the damping coefficients.

The final set of data, that covers a very important aspect of the entire analysis, is the inhibited degrees of freedom, i.e. degrees of freedom that are set to zero.

b) The second is related to the symbolic matrix calculus of the dynamic model and to the visualization of the mechanism;

After loading and defining the data to the simulator, by pressing the **Robot Looks** button the simulation begins. The purpose of this phase is to create and build all the necessary data to perform the dynamic analysis of the chosen spatial robot.

First of all the parameters are checked in order to evaluate their feasibility; after that the first and second order kinematics are computed; then, an iterative symbolic algorithm, based on the previously described ERLS formulation, allows to build the main matrices of the dynamic formulation; finally all the symbolic variables and created symbolic matrices such as Jacobian matrix, mass matrix, stiffness matrix, Coriolis and external forces matrices are saved.

The procedure is completely iterative and the constraint equations are automatically taken into account when constructing the matrices allowing to avoid the need of a new set of equations. Matrices are computed along the links chain starting from the chassis.

After the initial calculation of the characteristic matrices of the robot, the simulator plots the position of the entire mechanism as shown in Figure 2. The upper figure shows the robot with the main frames according to DH parameters while the bottom figure shows the robot where the local reference coordinate frames are highlighted**.**

c) The third part is related to the dynamic simulation and results evaluation.

The dynamics of the system are implemented and simulated thanks to the Simulink toolbox of Matlab^{TM}.

The time of simulation and the solver to use in Simulink are the two parameters that are introduced directly while external input forces or torques have to be loaded and defined in the Simulink environment. After that, the system in a static condition is evaluated in order to obtain the initial conditions of the system. Then, the real values are substituted into the symbolic part. Finally, by using and linking the suitable Matlab^{TM} functions in Simulink, the dynamic behavior of the robot is simulated and the results visualized and saved.

The simulator, as output, plots the displacement of the nodes of the system according to the time and also the trajectory followed both by the flexible-link robot and by the chosen ERLS.

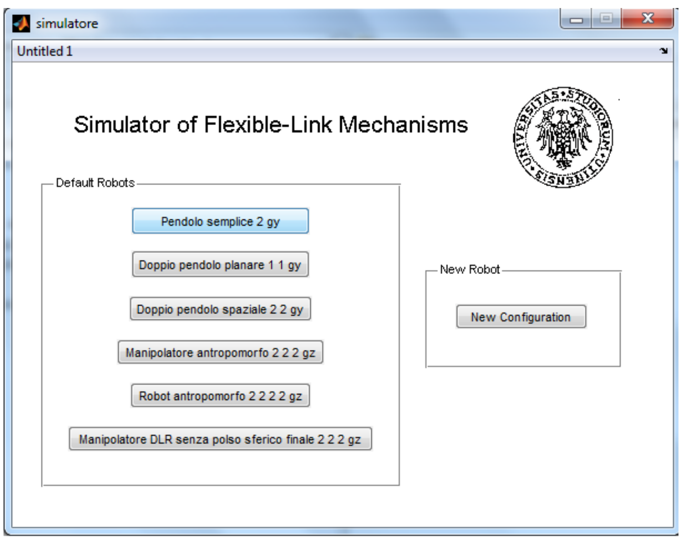

Figure 1. The first menu of the simulator

SIMULATION RESULTS

In order to show the capabilities of the simulator and to validate if, the ERLS simulation results are compared with those obtained by means of the Adams- Flex^{TM} software that exploits the FFR approach and models the flexible mechanisms by means of a component mode synthesis (CMS) technique based on the Craig-Bampton method [8].

In this paper, two different mechanisms have been considered as benchmarks:

a) The first mechanism considered as a benchmark is a three degrees of freedom anthropomorphic robot (Figure 3) which main kinematic, geometrical and mechanical parameters are shown in Table 1. It has three links end three revolute joints

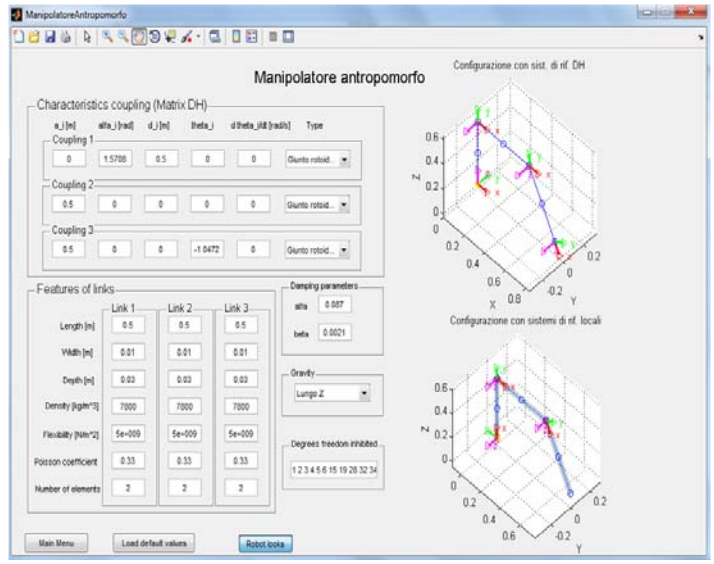

Figure 2. Starting position of the entire mechanism for anthropomorphic robot

The beam section is rectangular and external forces and torques are present as gravity effect and torque applied on the first joint. For each link of the mechanism, only two beam elements (thus three nodes) has been considered, so nine nodes and three rigid degrees of freedom (represented in Figure 4 as q1, q2 and q3) are present.

In order to fulfil the data requested, the fake degrees of freedom have to be considered and the related equations inposed. Let $s_x(i)$, $s_y(i)$, $s_z(i)$ be the X, Y and Z displacement of the i-th node and let $s_{rx}(i)$ and $s_{ry}(i)$ be the X and Y rotations, respectively, the compatibility equations impose:

$$\begin{aligned}
&s_x(1) = s_y(1) = s_z(1) = 0; s_{rx}(1) = s_{ry}(1) = 0 \\
&s_x^k(3) = s_x^k(4); s_y^k(3) = s_y^k(4); s_z^k(3) = s_z^k(4) \\
&s_{rx}^k(3) = s_{rx}^k(4); s_{ry}^k(3) = s_{ry}^k(4) \\
&s_x^k(6) = s_x^k(7); s_y^k(6) = s_y^k(7); s_z^k(6) = s_z^k(7) \\
&s_{rx}^k(6) = s_{rx}^k(7); s_{ry}^k(6) = s_{ry}^k(7)
\end{aligned} \tag{27}$$

Where **k** superscript refers to a generic common local frame. Now to be able to correctly define the ERLS, values of the elastic displacements of the three among the remaining degrees of freedom must be zero. In this case, three sets of degrees of freedom have been chosen:

$$s_{rz}^k(3) = 0; s_{rz}^k(4) = 0; s_{rz}^k(7) = 0 \tag{A}$$

$$s_{rz}^k(3) = 0; s_{rz}^k(4) = 0; s_{rz}^k(9) = 0 \tag{B}$$

$$s_{rz}^k(3) = 0; s_{rz}^k(6) = 0; s_{rz}^k(9) = 0 \tag{C}$$

in order to simulate the flexible-link mechanism with different equivalent rigid-link systems and show the effectiveness of a correct choice.

In the first simulation, Rayleigh damping coefficients are introduced as α=0.087 and β=0.0021, the Young's module is $9*10^9$(N/m^2) and the input torque is applied on the first joint. In this case, for zeroed degrees of freedom in Matlab™ software simulator, set of A has been chosen.

The Y coordinates of the tip of the second and third links of the anthropomorphic robot for the three selected sets of degrees of freedom have been plotted in Figures 4 and 5 where the simulation result of the three sets can be seen.

Table 1 Mechanism parameters

Link	a_i	α_i	d_i	θ_i	Length (m)	Width (m)	Depth (m)	Density (kg/m^3)	Poisson's Ratio
1	0	$\pi/2$	L_1	q_1	0.5	0.03	0.01	7800	0.33
2	L_2	0	0	q_2	0.5	0.01	0.03	7800	0.33
3	L_3	0	0	q_3	0.5	0.01	0.03	7800	0.33

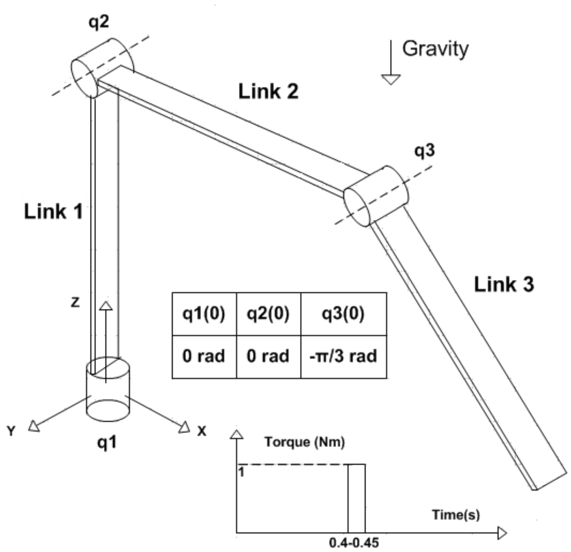

Figure 3Anthropomorphic robot

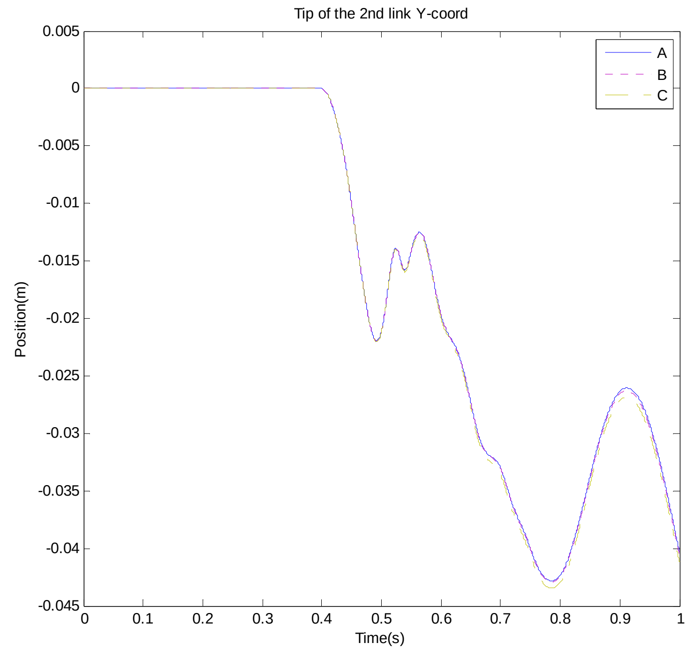

Figure 4Tip of the 2nd link Y-coord

A second simulation has been carried out by changing some parameters: the Rayleigh damping coefficients are set to α=0.087 and β=0.0021 and the Young's module to $5*10^{10}$(N/m^2). The input torque is applied on the first joint and its trend is the one shown in Fig.3. The zeroed degrees of freedom in Matlab™ software simulator have been chosen as the A set previously defined. The positions of the first and third links of anthropomorphic robot are compared with the results provided by the Adams™ software and plotted (Figures 6, 7, 8 and 9) showing a very good agreement both in amplitude and frequency.

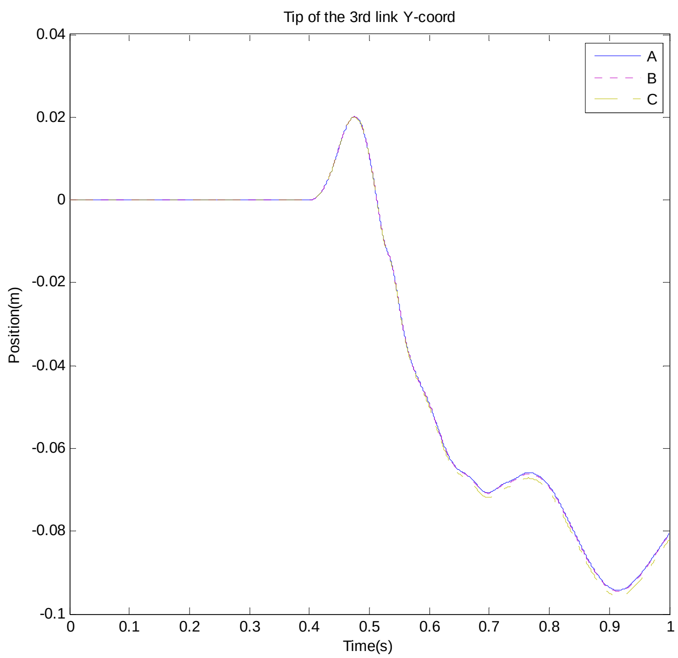

Figure 5 Tip of the 3rd link Y-coord

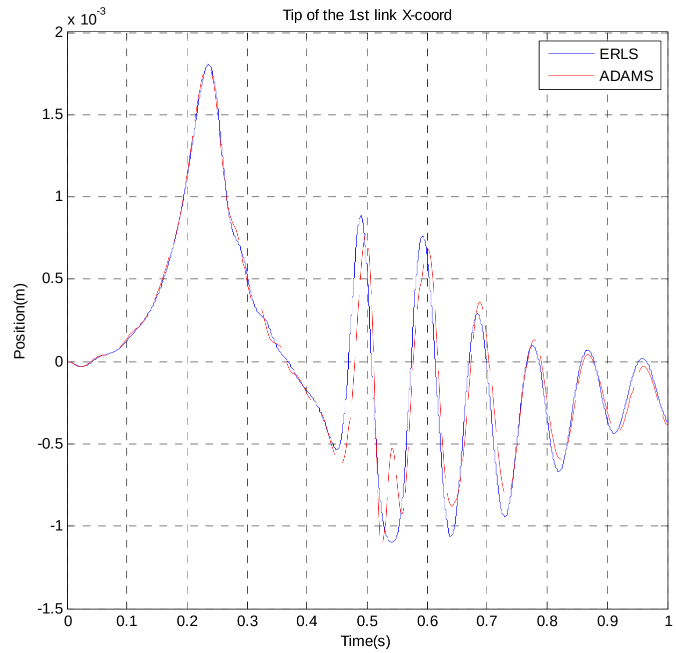

Figure 7Tip of the 1st link X-coord

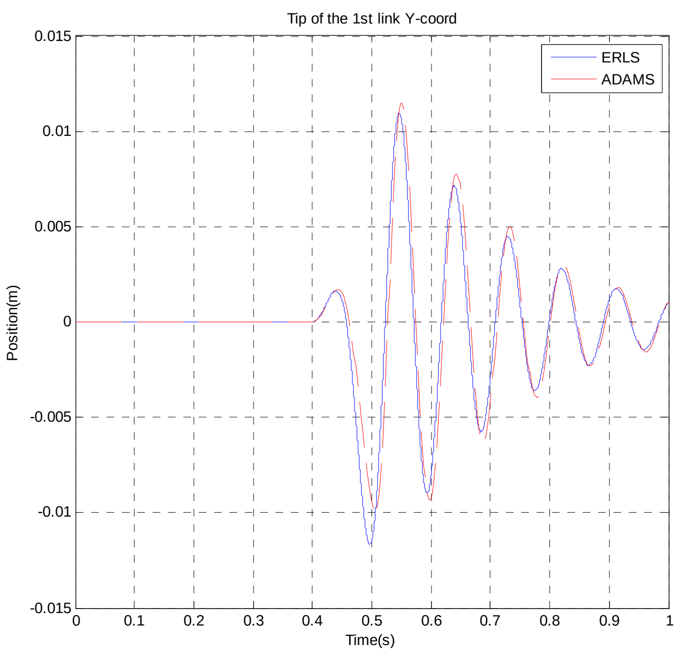

Figure 6 Tip of the 1st link Y-coord

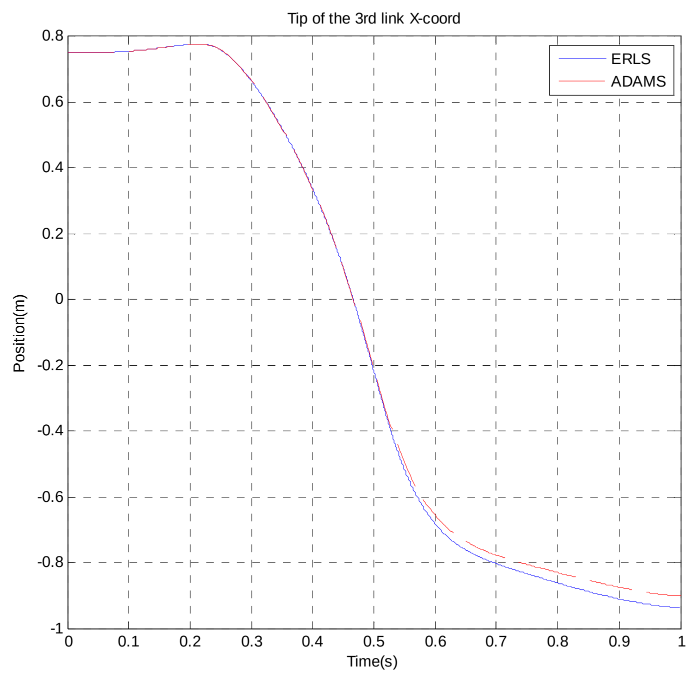

Figure 8 Tip of the 3rd link X-coord

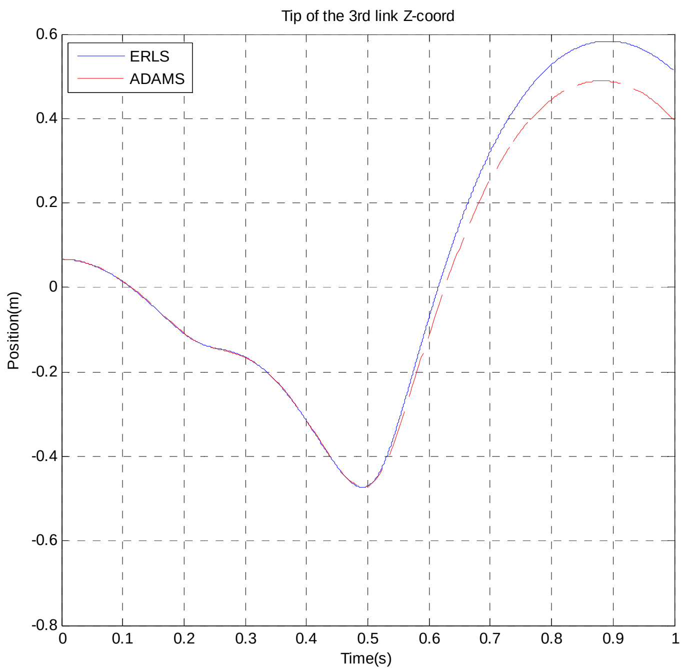

Figure 9 Tip of the 3rd link Z-coord

b) The second considered robot as a benchmark is a double planar robot (Figure 10). Its main kinematic, geometrical and mechanical parameters are shown in Table 2. For each link of the mechanism, one beam element (thus two nodes) has been considered, so four nodes and two rigid degrees of freedom (represented in Figure 10 as q1 and q2) are present. As can be seen, even if the system is planar, it is under spatial external forces and torques. Thus, the overall motion and vibrational effect is in 3D.

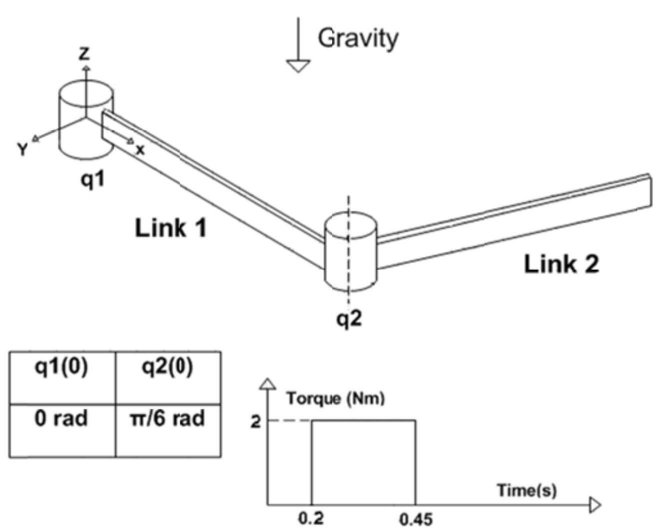

Figure 10 Double planar pendulum

Table 2 DH and mechanical parameters

Link	a_i	α_i	d_i	θ_i	Length (m)	Width (m)	Depth (m)	Density (kg/m^3)	Poisson's Ratio
1	L_1	0	0	q_1	0.5	0.03	0.01	7840	0.3
2	L_2	0	0	q_2	0.5	0.01	0.03	7840	0.3

And the compatibility equations impose:

$$
\begin{aligned}
&s_x(1) = s_y(1) = s_z(1) = 0; s_{rx}(1) = s_{ry}(1) = 0 \\
&s_x^k(2) = s_x^k(3); s_y^k(2) = s_y^k(3); s_z^k(2) = s_z^k(3) \\
&s_{rx}^k(2) = s_{rx}^k(3); s_{ry}^k(2) = s_{ry}^k(3)
\end{aligned}
\tag{28}
$$

In this case, it was chosen to set to zero the Y translation of node 2 and the Z rotation of node 3.

$$
s_y^k(2) = 0; s_{rz}^k(3) = 0 \tag{29}
$$

Rayleigh damping coefficients are α=2 and β=0.0005 and Young's module is $2*10^{11}$(N/m^2). The input torque is applied on the first joint and its trend is shown in Fig.10.

The extreme positions of the first and second links of the double planar pendulum are compared with Adams™ software results and plotted in Figures 11 and 12 (from up to down, X, Y and Z coordinates) and the results show very good agreement.

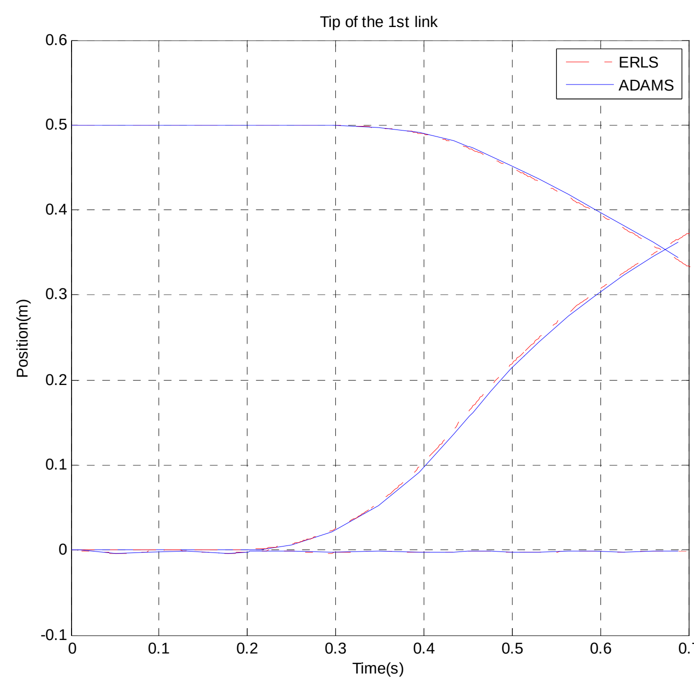

Figure 11Tip of the 1st link

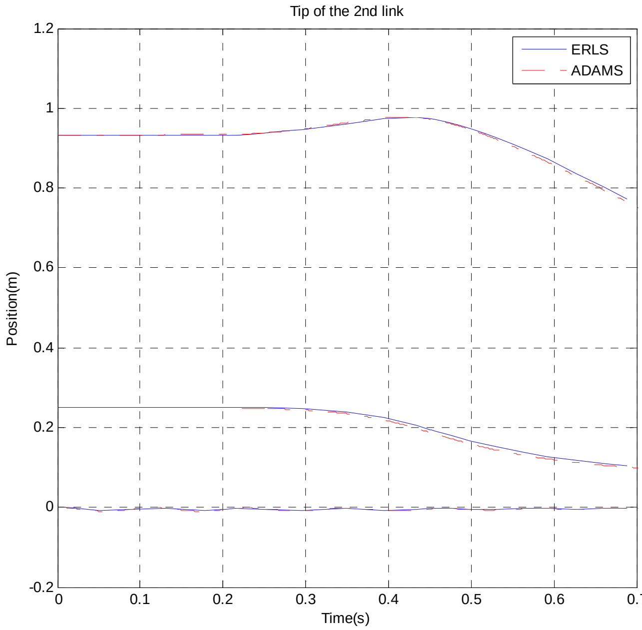

Figure 12Tip of the 2nd link

CONCLUSIONS

In this paper the dynamic formulation for flexible-link mechanisms based on an ERLS approach, where the basic idea is to decompose the overall motion of the mechanism into the rigid motion of a suitably defined ERLS and an overlapped elastic motion, has been evaluated. After the kinematic formulation, the equations of motion for the flexible mechanism have bene obtained by direct application of the virtual work principle.

A generic Matlab^{TM} software simulator that allow to simulate rigid-flexible-link systems, based on ERLS approach, has been implemented and is presented in this paper. Thanks to the ERLS based formulation, since it exploits the DH notation and the main concepts of the robotics kinematics, the approach to the flexible-link robots remains the same of the rigid ones allowing an easy approach. Then, in order to show the effectiveness of the method and of the simulator, different behaviors of specific robots with respect to different working conditions and mechanical parameters have been investigated.

The results have been compared with respect to Adams^{TM} showing a very good agreement (the small difference is because of defining ERLS) and, hence, the effectiveness of the method and the simulator.

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