On the Modeling of Flexible-Link Robots: First Experimental Validation of an ERLS-FEM Dynamic Model

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***Abstract*-The task of modellinG and control of lightweight robots is directly related to a suitable motion planning and control. To achieve such results and increase performances, accurate dynamic models that take into account the usually neglected inertial and elastic terms can be adopted in model-based approaches.**

In this paper, the experimental validation of an effective method based on an Equivalent Rigid Link System approach has been assessed. To this end, a dynamic simulator implementing the formulation has been exploited and an experimental test-bench has been set-up. The experimental tests carried out with a benchmark L-shape mechanism show a good agreement between the numerical model and the experimental measurements.

I. INTRODUCTION

In robotics, one of the main targets of the applied research is the dynamic performances improvement. Thus, the goal is to lighten the structure while maintaining a high degree of precision and accuracy. To this end, inertial and elastic effects have to be taken into account and controlled to avoid damages or unwanted effects such as arise of vibrations, premature wear and mechanical breaking. To limit or turn down such effects, different approaches can be adopted such as: (a) ad-hoc non model-based trajectory planning techniques [1,2].

Indeed, motion laws have to induce forces and torques at the joints compatible with the given constraints, thus reducing the possibility to excite mechanical resonance modes, i.e. a smooth trajectory has to be planned. As proved in [3-13], limiting the jerk reduces wear and turns down resonant frequencies excitation; (b) exploit effective dynamic models that include inertial and elastic effects in the motion planning and control. Indeed, the hypothesis of link/arm rigidity cannot be adopted when the structure is lightened and the velocity increased. Effective examples of this approach for flexible-link mechanisms can be found in [14-18].

In literature, different dynamic models have been propose for flexible-link robots but, if spatial high-dynamic systems are considered, there is still a large amount of interest especially from the experimental point of view [19-22].

In this attempt, a first experimental validation of a dynamic formulation for flexible-link mechanisms is carried out. The paper is organized as follows: the dynamic model is summarized in Section 2, in Section 3 the experimental set-up and numerical simulator are described and, in Section 4, the experimental model validation is presented and the results discussed.

II. THE EQUIVALENT RIGID LINK SYSTEM MODEL

The method under test is based on an Equivalent Rigid Link System (ERLS) concept [23]. It has been developed for accurate dynamic modeling of systems with large displacements and small elastic deformation by exploiting a Finite Element Method (FEM) link discretization.

The approach, here called ERLS-FEM [24-25], basically defines the elastic displacement with respect to an Equivalent Rigid Link Mechanism (ERLS). Each link is evaluated by means of spatial beam finite elements (FEM). In this approach, the mutual influence between gross body motion and vibration is taken into account, the kinematic equations of the ERLS are decoupled from the compatibility equations of the displacements at the joints, and spatial kinematics concepts such as the Denavith-Hartenberg notation can be adopted to formulate and solve the ERLS kinematics. Results can be used and easily integrated in the dynamic equations of the flexible multibody system under study.

1. Kinematics

Being {X, Y, Z} a fixed global reference frame, let $\mathbf{u}_i$ and $\mathbf{r}_i$ be the vector of the nodal elastic displacements of the i-th finite element and the vector of the nodal position and orientation for the i-th element of the ERLS respectively, and $\mathbf{w}_i$ and $\mathbf{v}_i$ be the position vector of the generic point of the i-th element of the ERLS and its elastic displacement respectively, the absolute nodal position and orientation of i-th finite element bi with respect to the global reference frame is:

$$b_i = r_i + u_i \tag{1}$$

and the absolute position pi of the generic point inside the i-th finite element is :

$$p_i = w_i + v_i \tag{2}$$

Let {xi, yi, zi} be a local reference frame, which follows the ERLS motion. It can be expressed with respect to the ERLS by means of a set of generalized coordinates **q**, the m-rigid

degrees of mobility of the mechanism, by exploiting the DH notation that can be adopted to describe the kinematics of the ERLS. The **r**i's can be gathered into a unique vector **r**, representing the position and orientation of the whole ERLS (Fig. 1).

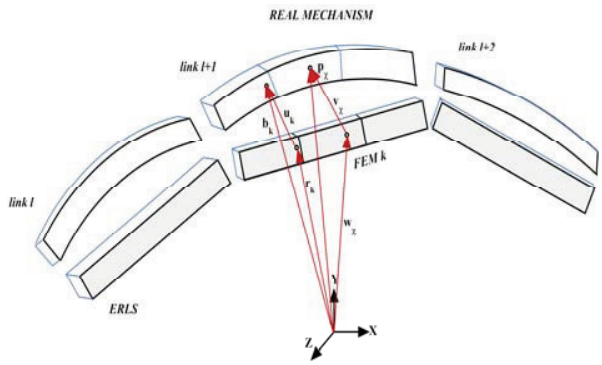

Fig. 1. Kinematic definition if the ERLS

By defining and exploiting a local to global transformation matrix $\mathbf{R}_i(\mathbf{q})$, a block-diagonal rotation matrix $\mathbf{T}_i(\mathbf{q})$ and an interpolation function matrix $\mathbf{N}_i(x_i,y_i,z_i)$, the virtual displacements in the fixed reference frame and the acceleration of a generic point inside the i-th finite element can be computed (see [24-25] for more details).

1. Dynamics

The dynamic equations are obtained by applying the principle of virtual work and computing the inertial, elastic and external generalized forces terms:

$$\begin{aligned}&\sum_i \int_{v_i} \delta p_i^T \ddot{p}_i \rho_i dv + \sum_i \int_{v_i} \delta \varepsilon_i^T D_i \varepsilon_i dv = \sum_i \int_{v_i} \delta p_i^T g \rho_i dv \\ &+(\delta u^T + \delta r^T) f \\ &\delta W^{inertia} + \delta W^{elastic} = -\delta W^{external}\end{aligned} \tag{3}$$

ρi, **D**i and **ϵ**i are the mass density, the stress-strain matrix and the strain vector for the i-th element respectively, **g** is the gravity acceleration vector and **f** is the vector of the external forces and torques; δu and δr refer to all nodes of the model.

Nodal elastic virtual displacements δ**u** and virtual displacements of the ERLS δ**r** are completely independent. Thus, two set of equilibrium equations, i.e. local nodal equilibrium and global equilibrium equations, can be obtained from Eq. (3) by zeroing alternatively the nodal elastic virtual displacements and the virtual displacements of the ERLS.

The following system of differential equations are obtained:

$$\begin{aligned}&M(\ddot{r} + \ddot{u}) + 2(M_{G1} + M_{G2})\dot{u} \\ &+(M_{C1} + 2M_{C2} + M_{C3})u + Ku = \left(f_g + f\right)\end{aligned} \tag{4}$$

$$\begin{aligned}&J^T M(\ddot{r} + \ddot{u}) + 2J^T (M_{G1} + M_{G2})\dot{u} \\ &+J^T (M_{C1} + 2M_{C2} + M_{C3})u = J^T \left(f_g + f\right)\end{aligned} \tag{5}$$

M is the mass matrix, $\mathbf{M}_{g1}$ and $\mathbf{M}_{g2}$ are the Coriolis terms, $\mathbf{M}_{c1}$, $\mathbf{M}_{c2}$ and $\mathbf{M}_{c3}$ the centrifugal stiffness terms, **K** the stiffness matrix, **J** the Jacobian matrix, and $\mathbf{f}_g$ the vector of the equivalent nodal loads due to gravity.

Dynamic equations, after the substitution of the second order differential kinematics equations of the ERLS, can be grouped and rearranged in matrix form, Eq. 6.

A Rayleigh model of damping has been considered and inserted in the model, i.e. α and β, to deal with realistic flexible manipulator systems.

By solving the system Eq. (6), accelerations can be computed and, by means of an appropriate integration scheme velocities and of displacements can be obtained (see [24-25] for more details).

$$\begin{bmatrix} M & MJ \\ J^T M & J^T MJ \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{q} \end{bmatrix} =$$

$$\begin{bmatrix} -2\left(M_{G1} + M_{G2}\right) - \alpha M - \beta K & -M\dot{J} & -\left(M_{C1} + 2M_{C2} + M_{C3}\right) - K \\ J^T\left(-2\left(M_{G1} + M_{G2}\right) - \alpha M\right) & -J^T M\dot{J} & -J^T\left(M_{C1} + 2M_{C2} + M_{C3}\right) \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{q} \\ q \end{bmatrix}$$

$$+\begin{bmatrix} M & I \\ J^T M & J^T \end{bmatrix} \begin{bmatrix} g \\ f \end{bmatrix} \tag{26}$$

III. Experimental and Simulation Set-Up

1. Numerical Simulator

According to the method, a generic Matlab software simulator has been developed: it exploits the DH notation and the main concepts of robotics kinematics, allowing to study the dynamics flexible-link robotic systems without differences with respect to the rigid ones (see Fig.2, [6]).

This simulator is structured in three main parts:

- DH, geometrical and mechanical parameters definition.
  Here, the main parameters are inserted to unambiguously define the mechanism.

- Symbolic matrix calculus of the dynamic model and visualization of the mechanism.
  In this phase parameters are checked, first and second order kinematics are computed, and an iterative symbolic algorithm is run to build and save the main matrices of the dynamic formulation. The mechanism initial configuration is then shown.

- Dynamic simulation and results evaluation. The Simulink Matlab toolbox is exploited to compute,

visualize and save the dynamic behavior of the robot; simulation time and solver can be directly chosen while external input forces or torques have to be defined in the Simulink environment.

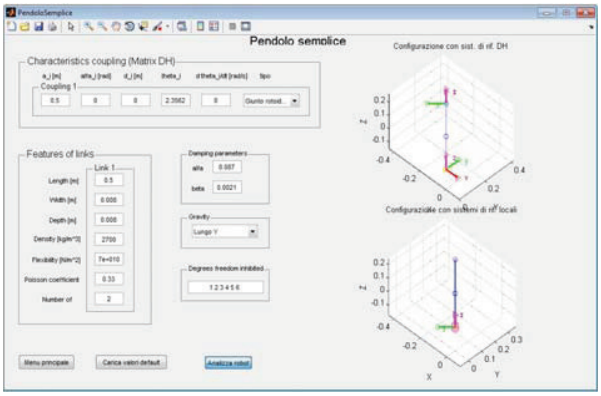

Fig. 2. Matlab numerical Simulator - GUI

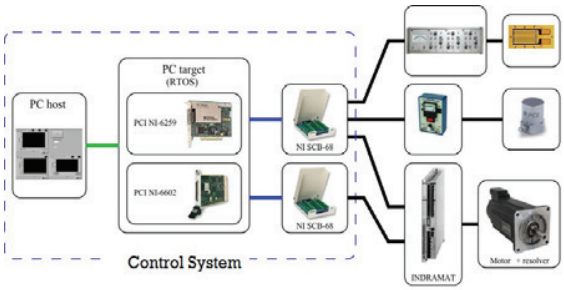

Fig. 3. Scheme of the experimental set-up

2. *Test-bench and experimental set-up*

In this section the experimental control and measurement set-up and the benchmark mechanism used for the tests are described.

Looking at Fig. 3, the different components and equipment can be described:

- PC Desktop (PC Target) equipped with:
  a. DAQ board NI 6259.
  b. Digital acquisition board NI 6602.
- Laptop (PC Host).
- Signal Amplifier HBM - KWS3082A and Wheatstone bridge for strain-gauges signal conditioning.
- Notor drive system Indramat DKC11.1-040-7 FW.
- SPM brushless motor MKD090B-047, with 12 Nm peak torque.
- Accelerometer PCB 352C65 and signal amplifier PCB 480E09.

In order to control the mechatronic system, a Real-Time operative system has to be exploited. The control software has been implemented by means of the National Instruments Labview 2010 software suite. In particular, the Real-Time module has been used on the Target PC for a Real-Time Operative System (RTOS). The Host PC, connected to the Target PC by means of an Ethernet connection, is devoted to remote control and data storage and processing. A user-friendly interface has been designed to remotely control the system. The RT process sampling frequency has been set to 1kHz. In order to drive the motor and excite the mechanism that is going to be tested, an open-loop torque control has been used.

The control and acquisition system have been tuned and calibrated and an L-shape benchmark mechanism has been designed and mounted to perform the model validation. The particular L-shape has been chosen to allow both a 3D motion, i.e. induce motion and vibrations in different planes, and a simple gross motion control of the mechanism.

The L-shape mechanism is shown in Fig. 4. As can be seen, a shrink disk connects the motor to the first half of the L, which is linked to the second half by means of a rigid elbow. The accelerometer is on this rigid connection while strain gauges are installed at the midspan of both links in an half-bridge configuration.

Fig. 4. Mechanism under test

The L-shape mechanism is made of two flexible rods of aluminum with a square section, connected by a rigid aluminum joint.

IV. EXPERIMENTAL VALIDATION

The torque signal that drives the motor is chosen to fast accelerate and decelerate the mechanism starting from a statically balanced configuration. Thus, the L-shape mechanism is driven by two steps in opposite directions (see Figure 5). Starting from 135°, the first step moves the mechanism for about 105° and then the second step and the remaining signal stops the motion at about 30°.

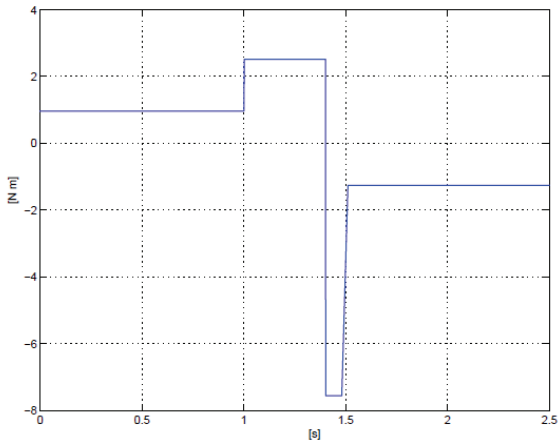

Fig. 5. Input signal

The mechanical parameters of the L-shape mechanical components are as follows:

- 1st beam: aluminium,L1=0.5m,cross section: 8×8mm
- 2ndbeam:aluminium,L2=0.5 m, cross section:8×8mm

The aluminium properties are:

- Density: 2700 kg/m^3
- Young's modulus 7e10 N/m^2
- Poisson's coefficient: 0.33

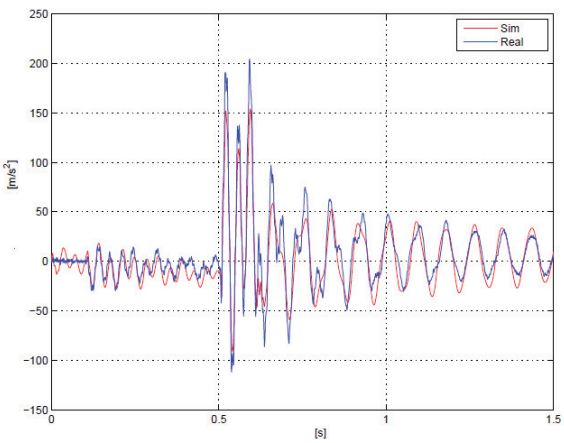

Fig. 6. Elbow Y-coord acceleration signal

The numerical simulations on the Matlab simulator have been carried out by considering two beam elements per each part of the L-shape mechanism and a Runge-Kutta solver has been chosen.

In Figure 6 the comparison between the simulated and measured acceleration in the Y coordinate of the last node of the first part of the mechanism, i.e. the elbow, is shown while in Figure 7 the frequency spectrum of the two signals are presented.

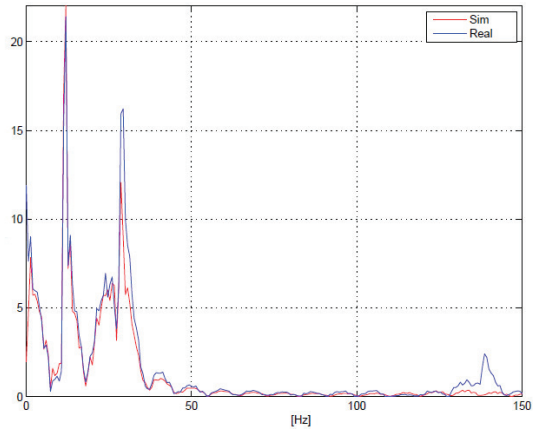

Fig. 7. Elbow Y-coord acceleration signal – frequency spectrum

In Figure 8 the bending moment in the Z local coordinate of the second link, result of the processing of the signal that come from the strain-gauges, is shown with respect to time.

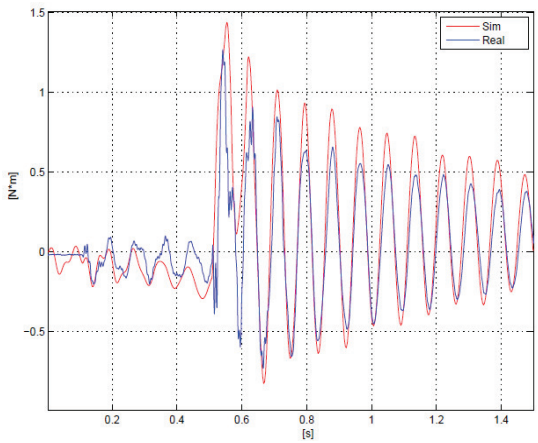

Fig. 8. Bending moment in local Z-coord of the second-half of the L-shape mechanism

By looking at the different experimental results, it can be appreciated that the simulated signals match very well the experimental ones in the time domain since they are almost overlapped. Looking at the plots in the frequency domain (Fig.7) again, the results show how the simulated and

experimental data are very similar. In particular, the main frequencies of the mechanisms are correctly modeled and captured. Signal amplitudes and vibrational behavior are captured even if only two beam elements per link have been considered in the simulated model.

V. CONCLUSIONS

In this work, the ERLS-FEM formulation for flexible-link robots has been considered for a first experimental validation.

An experimental set-up has been designed and built and a L-shape flexible-link mechanism has been tested. Acceleration and bending moment have been measured and the comparison between the numerical simulator results and the experimental data are compared and evaluated showing a good agreement between the two.

Future work will cover the experimental validation of the model for multi-link mechanisms and the integration of component mode synthesis techniques in the dynamic model.

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