Energy Efficiency and smoothness in robotics trajectory planning: numerical simulation and comparison

Paolo Boscariol[3], Andrea Gasparella[1], Alessandro Gasparetto[3], Nicola Lever[1], Dario Richiedei[2], Alberto Trevisani[2] and Renato Vidoni[1]

Abstract— **In this paper, the most widely adopted industrial off-line non model-based trajectories together with optimum time-jerk and time-energy algorithms are considered and evaluated in terms of energy efficiency and smoothness.**

First of all, a robotic dynamic simulator able to run different laws of motion, to simulate the robot dynamic behavior and to evaluate the amount of mechanical energy, torque and jerk, has been developed and implemented in Matlab.

After that, both point-to-point and pick-and-place trajectories have been simulated by comparing different motion laws whose results have been evaluated and ranked from both the energy efficiency and smoothness point of view.

Finally, a performance index able to take into account the energetic and vibrational performance has been defined to compare the different trajectory planning algorithms.

I. INTRODUCTION

Trajectory planning is a fundamental issue for robotics and mechatronics applications. Indeed, the ability to generate trajectories with prescribed features is a crux to ensure effective results in terms of quality and feasibility of performing the required motion. Different criteria aimed at optimizing the motion have been proposed in literature [1],[2] and attention has been mostly paid on performing fast motions and, eventually, ensuring adequate smoothness. In contrast, only few works address the optimization of the energy consumption, although energy-based optimal trajectory planning criteria can cover an important role in the frame of a green-mechatronic approach and sustainable vision.

Indeed, the concept of energy efficiency and conservation in automation industry and robotics has become in focus only in the recent years, due to the increasing of the energy costs and to the problems and rules fixed to limit or control the climate change. Thus, at today, the energy saving target is not just a mere economic implication, but also an ethical issue and a possible add-on for the market competitiveness of the industrial products and applications. Among the techniques to reduce energy consumption in robotic and mechatronics systems, the development of energy efficient trajectories shows promising results since it does not rely on hardware modifications and therefore can be easily implemented in both new and existing systems to improve their efficiency.

Besides energy efficiency optimization, for planning an effective trajectory other features have to be achieved. In particular, it has to be taken into account that severe vibrations can arise in manipulators when they are moved along a non-smooth trajectory. In that case worsening of accuracy, premature joint wear and mechanical failures might occur [3].

To test this purposes, in this work, the simultaneous evaluation of both the energy efficiency and the smoothness in off-line trajectory planning in robotics and, more in general, in industry is addressed.

In literature, extensive surveys on trajectory planning techniques can be found [1], [2], but rarely a comparative performance analysis and a performance index definition have been proposed (e.g. [4]). In general, a possible solution to accomplish a given task using a robotic manipulator is to synthesize the optimal motion with respect to a relevant criterion. Thus, focusing on generating off-line movements to perform tasks known a priori and in a defined environment, it is possible to find different optimality criteria based on the minimization of the execution time, actuator effort or jerk.

A fundamental distinction between the methods available in literature is the use of a model-based or of a model-free approach. Model-based approaches can achieve good results (e.g. [5], [6]) in specific cases but they lack of generality, which is a fundamental requirement for most industrial applications. As a matter of fact, usually, most industrial facilities are not adequately modeled to address model-based approaches, and the personnel training investment is not reputed to be profitable. Therefore model-free approaches, as the ones considered in this paper, are more appealing for todays market. Thus, the most significant off-line non-model based methods and algorithms currently adopted in industrial robotics and mechatronic systems are here considered. In particular, both the state-of-the-art trajectory planning algorithms such as trapezoidal and double-s methods [7], and ad-hoc developed methods with high orders of continuity or synthesized through optimization functions, are considered.

The investigated trajectory planning techniques are evaluated, compared and ranked both in terms of energy costs for clearly quantifying the possible performance enhancement and energy savings, and smoothness to evaluate the capability to provide fast motion while reducing low induced vibrations. Finally, a performance index synthesizing both energy efficiency and smoothness is proposed to provide a

*This work is supported by the Free University of Bolzano under the project TN5050 - Energy Efficiency Techniques for Mechatronic and Robotic Systems

[1] R. Vidoni, A. Gasparella and N. Lever are with the Faculty of Science and Technology, Free University of Bozen-Bolzano, Bolzano, Italy corr. author: renato.vidoni, at unibz.it

[2] D. Richiedei and A. Trevisani are with the DTG of Vicenza (I), University of Padua (I)

[3] P. Boscariol and A. Gasparetto are with the DIEGM of the University of Udine (I)

straightforward comparison of all the motion laws investigated.

II. DYNAMIC SIMULATOR

In order to perform the comparison of the different motion laws, an ad-hoc dynamic simulator for robotic systems has been implemented and developed in Matlab.

The dynamic simulator takes into account the geometrical and inertial parameters of the robot in use; in particular, a Newton-Euler approach has been implemented.

The simulator allows both selecting among classical rigid-link robots, e.g. Anthropomorphic and cartesian, or creating particular robots starting from a single-link configuration. Classical and ad-hoc trajectory primitives can be run and the system output are the position, velocity, acceleration and jerk profiles at the joints, and the actuator and system effort in terms of requested torques, work and power.

Thanks to this simulator, a straightforward evaluation of the required effort for each trajectory can be performed.

In Fig.1, the simulator user interface and an example of the simulator result experimental validation, i.e. comparison of the simulated and measured torque on a joint of a real robot, are shown.

Two robots have been chosen for the numerical evaluation: a cartesian robot and an anthropomorphic robot.

Tab. I, II and III report the main DH and mechanical parameters of the robots.

TABLE I

DH TABLE, CARTESIAN AND ANTHROPOMORPHIC ROBOTS

Cartesian	$\alpha[deg]$	$a[m]$	$\theta[deg]$	$d[m]$
Base frame	0	0	0	0.75
Joint1	90	0	0	d_1
Joint2	90	0	90	d_2
Joint3	0	0	0	d_3
Joint4	-90	0	θ_4	0
Joint5	90	0	θ_5	0
Joint6	0	0	θ_6	0.1
Anthrop.	$\alpha[deg]$	$a[m]$	$\theta[deg]$	$d[m]$
Base frame	0	0	0	0.75
Joint1	90	0	θ_1	0
Joint2	0	0.71	θ_2	0
Joint3	90	0	θ_3	0
Joint4	-90	0	θ_4	0.859
Joint5	90	0	θ_5	0
Joint6	0	0	θ_6	0.1

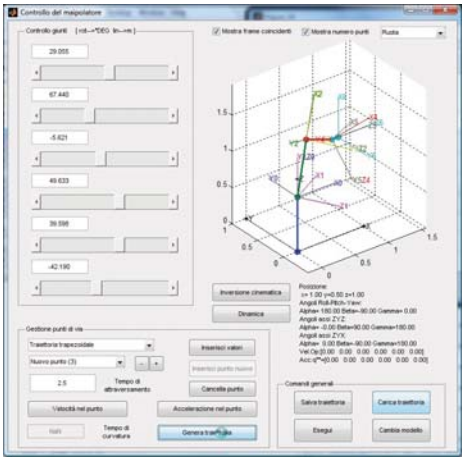
TABLE II

ROBOT ARMS MASSES

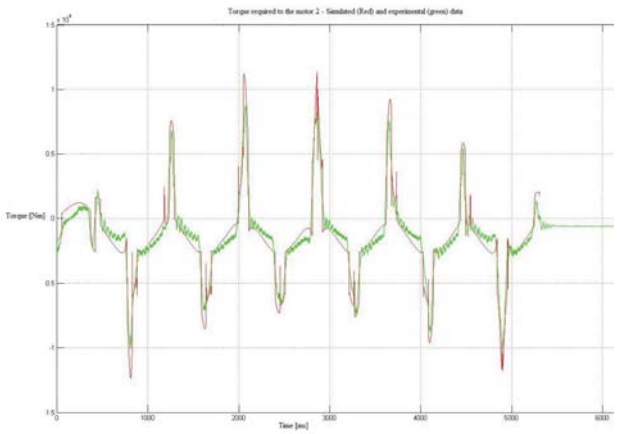
Arm	*base*	1	2	3	4	5	6
Cartesian [kg]	10	10	10	10	10	10	10
Anthrop. [kg]	190	45	45	40	18	8	4

III. PRIMITIVE TRAJECTORIES

Different industrial relevant robots and paths have been evaluated in order to effectively compare the motion laws.



(a) Dynamic simulator



(b) Simulator validation

Fig. 1. Dynamic simulator: a- user interface; b- validation

As far as robot type is concerned, three main systems have been simulated: a single-link single-motor system, an Anthropomorphic robot and a Cartesian robot. As for the paths both point-to point and complex path motions have been considered.

As regards as the *industrial trajectory* primitives, the following and most exploited methods have been evaluated [7]:

- Trapezoidal (T), linear trajectory with parabolic blends (three segments).
- Double-S (2S), trajectory with double S velocity profile (seven segments).
- Harmonic (H), trigonometric trajectory with an acceleration profile proportional to the position profile with opposite sign.
- Cycloidal (C), trigonometric trajectory with a continuous acceleration profile.
- Quadratic poly (PQ), parabolic trajectory.
- Cubic poly (P3), polynomial trajectory of degree three, four parameters.
- Quintic poly (P5), polynomial trajectory of degree five,

TABLE III

INERTIA MATRIX AND CENTER OF GRAVITY (COG) FOR EACH ARM OF THE ANTHROPOMORPHIC ROBOT

Arm	$base$	1	2	3
Inertia matrix $[kg \cdot m^2]$	$\begin{pmatrix} 70.59 & 0 & 0 \\ 0 & 70.59 & 0 \\ 0 & 0 & 18.12 \end{pmatrix}$	$\begin{pmatrix} 2.60 & 0 & 0 \\ 0 & 1.71 & 2.67 \\ 0 & 2.67 & 1.98 \end{pmatrix}$	$\begin{pmatrix} 0.84 & 0 & -9.09 \\ 0 & 10.56 & 0 \\ -9.09 & 0 & 9.91 \end{pmatrix}$	$\begin{pmatrix} 1.75 & 0 & 0.51 \\ 0 & 2.05 & -1.04 \\ 0.51 & -1.04 & 1.19 \end{pmatrix}$
CoG [m]	$\begin{pmatrix} 0 & 0 & -0.55 \end{pmatrix}$	$\begin{pmatrix} 0 & -0.125 & 0 \end{pmatrix}$	$\begin{pmatrix} -0.36 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0.08 & 0 & 0.09 \end{pmatrix}$
Arm	4	5	6	
Inertia matrix $[kg \cdot m^2]$	$\begin{pmatrix} 0.66 & 0 & 0 \\ 0 & 0.03 & 0 \\ 0 & 0 & 0.65 \end{pmatrix}$	$\begin{pmatrix} 0.016 & 0 & 0 \\ 0 & 0.012 & 9.95 \\ 0 & 9.95 & 0.007 \end{pmatrix}$	$\begin{pmatrix} 0.004 & 0 & 0 \\ 0 & 0.004 & 0 \\ 0 & 0 & 0.0006 \end{pmatrix}$	
CoG [m]	$\begin{pmatrix} 0 & 0.16 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -0.003 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -0.027 \end{pmatrix}$	

six parameters.

- SPLINE, spline trajectory with cubic primitive.

Among the *optimum trajectory* planning techniques, the following trajectory primitives have been implemented and simulated:

- Minimum acceleration (effort), trajectory that minimizes the integral of the square value of the joint accelerations,
- Continuous-jerk 445 [8],
- Minimum time-jerk minS3 [1], [4], this algorithm is based on cubic splines.
- Minimum time-jerk minBS5 [1], [4], this algorithm is based on quintic splines

In the Minimum time-jerk trajectories, either the parameters are properly chosen or the trajectory time is scaled or elongated through a time scale process to obtain the desired motion time. Moreover, these two optimum time-jerk trajectories, minS3 and minBS5, based on the minimization of a two-term objective function, have been simulated to evaluate their effectiveness in terms of energy and energy-jerk efficiency. The jerk contribute is taken into account in the minimization function as the integral of its squared value.

As previously stated, only off-line non-model based methods have been considered since their synthesis is independent from the particular robotic system under investigation and does not rely on the knowledge of any dynamic parameter or model of the system.

IV. COMPARISON

The first test has been made by considering a point to point motion along the X-axis of a cartesian robot, i.e. a single-link, single-motor system; the total displacement was 0.5 m. The gravity acceleration, 9.81 m/s^2, has been simulated on the Z-axis while friction has been considered in its static and dynamic effect. Tab. IV shows the numerical results.

The $T_{1/2}$ and $T_{1/3}$ trajectories are symmetric trapezoidal trajectories with the λ parameter, which defines the acceleration time, set to 1/2 and 1/3 of the motion time respectively.

The comparison of the results shows that, for a point-to-point linear motion, the P3 trajectory is the most efficient in terms of energy, measured through W. This can be viewed as a confirmation of the properties of this polynomial primitive

TABLE IV

POINT TO POINT MOTION WITH ZERO INITIAL AND FINAL VELOCITIES AND ACCELERATIONS FOR A SINGLE-LINK SINGLE-MOTOR SYSTEM - MOTION TIME = 6 S; P = POWER, W = WORK, τ = TORQUE, J = JERK

	$T_{1/2}$	$T_{1/3}$	2S	H	C	P3	P5
P_{RMS}	57.45	39.39	42.89	34.08	61.78	**30.89**	52.74
W	11.51	9.87	10.15	9.45	13.09	**8.98**	12.04
τ_{RMS}	100.00	92.09	102.21	87.66	110.52	**87.46**	102.99
$\int J^2$	∞	∞	411.08	∞	195.01	∞	**180.72**
J_{RMS}	∞	∞	20.27	∞	13.96	∞	**13.44**

that minimizes the quality index $\int_0^{t_f} \tau^2 dt$, where t_f is the total time of the trajectory.

Since if the jerk is limited or minimized the tracking accuracy increases and the excitation of the resonant frequencies is reduced, this value has an important significance in the trajectory algorithm performance.

As far as the jerk is concerned, only three trajectories show a finite value. Among the motion laws with finite jerk, the 2S trajectory is the less energy expensive, providing an increase of the 13% compared with the energy required by the P3.

The second test has been made by performing a pick and place motion with both a Cartesian and an Anthropomorphic robot. The same motion time has been considered for all the motion laws. The points to follow in the operative space are reported in Tab. V.

TABLE V

POINTS OF THE PICK AND PLACE MOTION IN THE CARTESIAN SPACE

Point	X [m]	Y [m]	Z [m]
P0	0.5	0.5	0.5
P1	0.5	0.5	0.25
P2	0.5	0.5	0.5
P3	0.2	0.1	0.5
P4	0.2	0.1	0.25
P5	0.2	0.1	0.5
P6	0.5	0.5	0.5

Tab. VI shows the corresponding joint values, solution of the inverse kinematics for the Anthropomorphic robot.

As can be seen, the angular values of two of the wrist joints remain constant along the whole trajectory.

TABLE VI

POINTS OF THE PICK AND PLACE MOTION IN THE JOINT SPACE FOR THE ANTHROPOMORPHIC ROBOT

Point	J1 [deg]	J2 [deg]	J3 [deg]	J4 [deg]	J5 [deg]	J6 [deg]
P0	45	61.69	-36.15	180	25.54	225
P1	45	38.82	-28.50	180	10.32	225
P2	45	61.69	-36.15	180	25.54	225
P3	26.57	81.09	-73.49	180	7.61	225
P4	26.57	31.30	-57.78	180	-26.48	225
P5	26.57	81.09	-73.49	180	7.61	225
P6	45	61.69	-36.15	180	25.54	225

Tab. VII and Tab. VIII report the simulation results in terms of energy and smoothness parameters.

Even if the performed motion is a pick and place, the movements made by the joints of the two robots are substantially different. Indeed, in the case of a cartesian robot, the main joint movements are point-to-point thus no continuous motion along the linear joints is planned. Indeed, if the motion along the Z-axis is considered, only the vertical linear joint is in charge to perform the action. On the contrary, in the Anthropomorphic robot, some joints have to move along the whole trajectory.

Thanks to this consideration, the results in Tab. VII and Tab. VIII can be better understood.

For the Cartesian robot movement (see Tab. VII), the P3 trajectory allows again the best result in terms of required power, work and torque. Among the optimal trajectories, it can be appreciated how the minimum time-jerk trajectories allow the best performances both in work and in jerk content.

If smoothness is also accounted for, the 2S represents, among the other industrial trajectories, the best compromise since it allows a good behavior in terms of energetic performances and a finite jerk RMS value.

As for the Anthropomorphic robot, the analysis of Tab. VIII leads to different considerations: the minimum time-jerk trajectories allow the best performance both for the energy and jerk parameters.

In both the simulations, the worst "energy" case is represented by the 445 law. This result can be easily justified since the overall path length to be run by the robot has to be noticeably increased to allow the smoothness required by the trajectory algorithm.

In Tab. IX the laws of motion are ranked and the deviation with respect to the best one is given in percentage for the Anthropomorphic robot.

As can be appreciated from the results, important savings and performance enhancements can be achieved by implementing the proper law of motion. Indeed, even if the torque requirements does not show important deviation from the minimum value, i.e. the minBS5, both the work, W, and jerk, J_{RMS}, values show great differences. As an example, the minS3 trajectory results the best choice in terms of energy and allows, for the simulated path, a reduction of more than the 10% with respect to a classical and widely adopted SPLINE, while providing a jerk finite value.

TABLE IX

TRAJECTORY RANK

Position	W		τ_{RMS}		J_{RMS}	
	Traj	Inv	Traj	Inv	Traj	Inv
1	minS3	-	minBS5	-	minBS5	-
2	P3	0.3%	P3	2.1%	minS3	14%
3	H	1.0%	mins3acc	2.2%	SPLINE	39%
4	T1/3	1.4%	H	2.3%	445	59%
5	2S	2.2%	T1/3	2.7%	P5	220%
6	T1/2	3.8%	minS3	3.1%	C	233%
7	P5	4.1%	SPLINE	3.1%	2S	384%
8	C	5.1%	2S	3.4%	minS3acc	1455%
9	minBS5	9.8%	T0.5	3.4%	-	-
10	minS3acc	10.2%	P5	4.2%	-	-
11	SPLINE	10.5%	445	4.8%	-	-
12	445	43.4%	C	5.0%	-	-

It can be added that, as a general remark, the optimum methods allow the best results both in terms of energy and jerk, and should be preferred when no point-to-point motions have to be performed. On the contrary, when point-to-point movements are requested along the trajectory, the effect of the optimization is reduced and the "classical" trajectory algorithms that allow a finite jerk value show a good compromise in terms of algorithm complexity and performances.

A. PERFORMANCE INDEX

In order to define and propose a synthetic criteria to classify the performance of a trajectory by taking into account both the energetic and vibrational requirements, a performance index (PI) has been defined.

The 2S trajectory has been chosen as the reference law due to its main characteristics: simplicity, industrial implementation and continuity in acceleration, hence finite jerk value.

The chosen PI takes into account the weighted relative values achieved by the considered trajectory in terms of energy and jerk with respect to the reference ones:

$$PI = k_e * \frac{W_i}{W_{ref}} + k_j * \frac{J_i}{J_{ref}}$$

where k_e+k_j=1. By setting to zero one of the two weights, the motion laws are classified either with respect to the energy efficiency or to the minimum jerk.

In order to be able to compare all the simulated trajectories, thus have a finite PI also for the laws with discontinuous acceleration, infinite jerk values have been included in PI by replacing them in the J_{RMS} with a high but finite upper-limit jerk value. This means that for each infinite peak a finite value has been accounted for a prescribed duration, i.e. $500m/s^3$ for 5 ms.

In this way, all the motion laws can be evaluated and compared on a same benchmark path, e.g. a pick and place or a smooth circular paths, allowing a direct comparison in terms of energy efficiency, smoothness or their combination with respect to the 2S standard law.

TABLE VII

CARTESIAN ROBOT - PICK AND PLACE; MOTION TIME = 6 S

	$T_{1/2}$	$T_{1/3}$	2S	H	C	P3	P5	SPLINE	445	minS3	minBS5
P_{RMS}	141.34	135.24	139.17	134.35	149.96	132.35	145.68	143.96	222.21	145.19	165.66
W	70.83	70.24	70.93	70.29	72.92	69.95	71.86	75.29	97.53	64.95	77.35
τ_{1RMS}	590.62	589.31	590.63	590.12	591.15	590.09	590.84	588.69	591.96	588.43	589.41
τ_{2RMS}	47.06	43.47	48.20	41.41	51.80	40.69	48.43	36.23	45.93	23.78	25.92
τ_{3RMS}	28.28	26.13	28.96	24.90	31.10	24.47	29.09	21.91	27.65	14.33	15.62
τ_{totRMS}	665.96	658.91	667.79	656.43	674.05	655.25	668.36	646.83	665.54	626.54	630.95
$\int J^2$	∞	∞	1231.96	∞	584.55	∞	540.23	114.35	204.62	70.19	50.40
J_{RMS}	∞	∞	14.33	∞	9.87	∞	9.48	4.36	5.84	3.42	2.89

TABLE VIII

ANTHROPOMORPHIC ROBOT - PICK AND PLACE; MOTION TIME = 6 S

	$T_{1/2}$	$T_{1/3}$	2S	H	C	P3	P5	SPLINE	445	minS3	minBS5	minS3acc
P_{RMS}	278.87	262.17	270.37	259.32	297.53	254.68	286.41	268.10	311.93	240.57	260.44	222.88
W	128.60	125.65	126.69	125.12	130.30	124.33	129.02	136.99	177.73	123.93	136.02	136.62
τ_{1RMS}	13.79	13.29	15.19	12.75	15.69	12.78	14.77	10.85	14.22	9.87	10.18	10.05
τ_{2RMS}	434.16	431.57	434.83	430.39	439.10	429.67	436.50	428.79	435.35	421.72	414.73	415.68
τ_{3RMS}	98.88	98.02	99.03	97.67	100.21	97.50	99.44	105.64	104.25	113.50	103.72	114.48
τ_{5RMS}	0.33	0.56	0.59	0.54	0.64	0.53	0.62	0.55	0.73	0.55	0.57	0.54
τ_{totRMS}	547.16	543.44	546.96	541.36	555.65	540.51	551.33	545.84	554.55	525.64	529.2	540.77
$\int J^2$	∞	∞	2.34×10^7	∞	1.11×10^7	∞	1.02×10^7	1.93×10^6	2.52×10^6	1.29×10^6	9.99×10^5	2.42×10^8
J_{RMS}	∞	∞	1976.12	∞	1361.15	∞	1308.48	567.72	648.12	465.31	408.37	6352.12

TABLE X

CARTESIAN ROBOT - PICK AND PLACE - PI

		$T_{1/2}$	$T_{1/3}$	2S	H	C	P3	P5	SPLINE	445	minS3	minBS5
W		70.8	70.2	70.9	70.3	72.9	70.0	71.9	75.3	97.5	64.9	77.3
J_{RMS}		76	76	14.3	50.1	9.9	58.0	9.5	4.4	5.8	3.4	2.9
W_i/W_r		1.00	0.99	1.00	0.99	1.03	0.99	1.01	1.06	1.38	0.92	1.09
J_i/J_r		5.30	5.30	1.00	3.50	0.69	4.05	0.66	0.30	0.41	0.24	0.20
n_{peak}		28	28	0	12	0	12	0	0	0	0	0
w_e	w_j											
0.2	0.8	4.44	4.44	1.00	3.00	0.76	3.44	0.73	0.46	0.60	**0.37**	**0.38**
0.5	0.5	3.15	3.15	1.00	2.24	0.86	2.52	0.84	0.68	0.89	**0.58**	**0.65**
0.9	0.1	1.43	1.42	1.00	1.24	0.99	1.29	**0.98**	0.99	1.28	**0.85**	1.00

TABLE XI

ANTHROPOMORPHIC ROBOT - PICK AND PLACE - PI

		$T_{1/2}$	$T_{1/3}$	2S	H	C	P3	P5	SPLINE	445	minS3	minBS5	minS3acc
W		128.6	125.65	126.69	125.12	130.3	124.33	129.02	136.99	177.73	123.93	136.02	136.62
J_{RMS}		10309	10309	1976	6251	1361	6271	1308	568	648	465	408	6352
W_i/W_r		1.02	0.99	1.00	0.99	1.03	0.98	1.02	1.08	1.40	0.98	1.07	1.08
J_i/J_r		5.22	5.22	1.00	3.16	0.69	3.17	0.66	0.29	0.33	0.24	0.21	3.21
n_{peak}		28	28	0	12	0	12	0	0	0	0	0	0
w_e	w_j												
0.2	0.8	4.38	4.37	1.00	2.73	0.76	2.73	0.73	0.45	0.54	**0.38**	**0.38**	**2.79**
0.5	0.5	3.12	3.10	1.00	2.08	0.86	2.08	0.84	0.68	0.87	**0.61**	**0.64**	**2.15**
0.9	0.1	1.44	1.41	1.00	1.21	0.99	1.20	**0.98**	1.00	1.30	**0.90**	0.99	**1.29**

Tab. X and Tab. XI show the results for different sets of weights for the pick and place trajectory for the Cartesian and Anthropomorphic robots.

The best PI are highlighted in the two tables.

Thanks to the defined PI it is possible to have a direct comprehension of the effectiveness of the chosen trajectory and its possible benefits in terms of performance, if any. If the case with weights equal to 0.5 is considered, the minS3 and minBS5 have the smallest PI while the most implemented industrial trajectories show a very high PI.

V. CONCLUSIONS

In this work the most adopted industrial trajectory planning techniques together with some minimum acceleration and jerk approaches have been considered and compared from an energy-smoothness performance point of view. Different trajectories have been simulated by means of an ad-hoc dynamic simulator and, after that, compared and ranked.

The results show that important savings in terms of energy can be achieved if the proper law of motion is selected, in particular if the minimization of the jerk content is considered as a performance improvement factor.

An energy-jerk performance index has been also defined in order to directly compare the different trajectory algorithms with respect to a classical double-S assumed as the reference.

Future work will cover the evaluation of the possible energy savings with respect to the kind of robot together with the quantification of the possible energy savings by regenerative braking systems.

REFERENCES

[1] A. Gaspaertto, P. Boscariol, A. Lanzutti, R. Vidoni, Trajectory Planning in Robotics, Mathematics in Computer Science, 2012, DOI 10.1007/s11786-012-0123-8

[2] AA Ata, OPTIMAL TRAJECTORY PLANNING OF MANIPULATORS: A REVIEW, Journal of Engineering Science and Technology, 2 (1), 32-54, 2007

[3] P.J. Barre, R. Bearee, P. Borne, E. Dumetz, Influence of a jerk controlled movement law on the vibratory behaviour of high-dynamics systems, Journal of Intelligent and Robotic Systems, 42(3):27593, 2005.

[4] A. Gasparetto, A. Lanzutti, R. Vidoni, V. Zanotto, Experimental validation and comparative analysis of optimal time-jerk algorithms for trajectory planning, Robotics and Computer-Integrated Manufacturing, 28, 164181, 2012

[5] Z. Shiller, Time-energy optimal control of articulated systems with geometric path constraints, J. Dyn. Syst. Meas. Control 11(8), 139143, 1996

[6] B.J. Martin, J.E. Bobrow, Minimum effort motions for open chain manipulators with task-dependent end-effector constraints. Int. J. Robot. Res. 18(2), 213224, 1999.

[7] L. Biagiotti and C. Melchiorri, Trajectory Planning for Automatic Machines and Robots, Springer-Verlag Berlin Heidelberg, 2010.

[8] K Petrinec, Z. Kovacic, Trajectory planning algorithm based on the continuity of jerk, In Control and Automation, 2007. MED07. Mediterranean Conference on, IEEE, pp. 15, 2007