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Robust trajectory planning for flexible robots

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Abstract

High speed operation is a recurring target in design and application of robotic manipulators. Moreover, maximizing the ratio between the weight of the payload and of the whole mechanism is a common objective. Therefore the use of a lightweight robot is a good practice that can help to reach these goals. On the other hand, the use of lightweight and therefore flexible manipulators requires the use of efficient control techniques and clever trajectory planning strategies [1]. For this reason, a large number of techniques have been proposed in literature to solve the problem of trajectory planning and control of such mechanisms [2]. Limiting our investigation only to the development of trajectory planning alogrithms, both model-based [3] and model-free techniques have been proposed [4].

In this paper we deal with the model-based trajectory planning of flexible-link robots. A large number of techniques have been developed for rigid-link robots, while the same problem is less frequently investigated for the case of manipulators with flexible-links, i.e. when the flexibility is distributed along the links of the robot. At the same time, the aim of this paper is to use the technique of desensitization to increase the robustness of the planned trajectory with respect to parametric perturbation of the plant. In fact several authors have emphasized [5] that the optimal control techniques which are commonly used in the case of model-based approaches, lead to a lack of robustness. This means that a trajectory that is optimal in the nominal case, is far from the optimal solution if applied to a perturbed plant. This can happen quite frequently, given the general difficulty of using (and tuning) accurate dynamic models of flexible-link mechanisms.

In this paper, a possible solution to this problem is proposed. The approach is based on the definition of a Two-Point Boundary Value Problem (TP-BPV), which is augmented with the introduction of one or more sensitivity functions. A similar approach has been used in some works by Singh [6], but its application is limited to linear plants. Here, a nonlinear model of a single flexible-link mechanism, which has been validated in [7], is taken into account.

The target is to develop an optimal robust rest-to rest trajectory for a single link robot that minimizes the control effort, i.e. a minimum torque trajectory. Other possible choices can be made by introducing in the cost function other quantities such as joint speed and elastic displacements.

The non-robust optimal trajectory problem is stated as follows. First of all, the dynamic model of the flexible-link robot is described by the system of ordinary differential equations (ODE): $\Phi(\mathbf{x}, u, t)$, being $\mathbf{x}$ the state of the system, u the control (i.e. the motor torque) and t the time. The Hamiltonian of the system is defined as: $\mathcal{H} = f + \lambda^T \Phi(\mathbf{x}, u, t)$. $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_N]$ is the vector of the lagrangians of the system, which has size N, i.e. the number of equations used in $\Phi(\mathbf{x}, u, t)$; f is the cost function that is optimized by the optimal control problem. The optimal control can be computed by solving the equation $\frac{\partial H}{\partial u} = 0$. By substituting the optimal control u^* in the Hamiltonian $\mathcal{H}$, we obtain $\mathcal{H}^*$. The latter can be differentiated in terms of $\mathbf{x}$ and λ leading to the new system of differential equations in the form:

$$\dot{\mathbf{y}} = \begin{bmatrix} \frac{\partial \mathcal{H}^*}{\partial \lambda} \\ -\frac{\partial \mathcal{H}^*}{\partial x} \end{bmatrix} \tag{1}$$

By using the Pontryagin minimum principle (PMP), the optimal trajectory of the state can be solved by establishing a TPBVP, i.e. by imposing the value of the state $\mathbf{x}$ at the initial point of the trajectory, and at the final point. In this case a rest to rest trajectory can be specified by imposing null initial and final velocities, and arbitrary positions at the initial and final point of the trajectory. The numerical solution of this problem is obtained through a collocation routine, implemented by Matlab command "bvp4c".

The robust solution is obtained in a similar manner, by introducing one or more sensitivity functions in the ODE system that describes the robot. In the case under investigation, the goal is to introduce the robustness of the trajectory planning algorithm to the change of the elastic constant k of the link. Therefore, we can add to $\Phi(\mathbf{x}, u, t)$ the partial derivative $S = \frac{\partial \Phi}{\partial k}$, obtaining therefore the new system of equations $\Phi_R = \begin{bmatrix} \Phi \\ S \end{bmatrix}$. Now the new "robust" system $\Phi_R(\mathbf{x}, u, t)$ can be used to define a new Hamiltonian $\mathcal{H}_R$, and a new system of differential equations, using the same procedure introduced above:

$$\dot{\mathbf{y}}_R = \begin{bmatrix} \frac{\partial \mathcal{H}_R}{\partial \lambda_R} \\ -\frac{\partial \mathcal{H}_R}{\partial x_R} \end{bmatrix} \tag{2}$$

Parametric robustness can be introduced by adding the boundary conditions that the sensitivity function S must be equal to zero both at the beginning and at the end-point of the trajectory. This techniques is called "desensitization" [6].

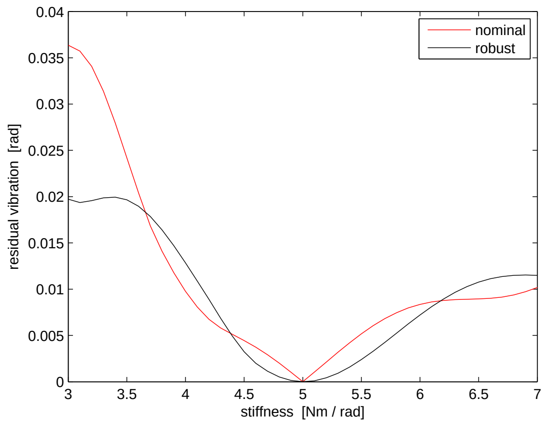

Figure 1. Amplitude of residual vibrations vs. the stiffness constant k

The two procedures described above have been compared by feeding the optimal control trajectory both to a nominal plant and to perturbed plants, i.e. to mechanisms that present a different elastic constant k. Results are shown in Figure 2 in terms of the peak amplitude of residual vibrations, i.e. the peak amplitude of the elastic displacements that happens in the link after that a rotation equal to $\pi/2$ is performed in 5 seconds. Figure 1 shows that for a nominal plant the residual vibration is equal to zero for both plants, and that the robust trajectory behaves better than the non-robust trajectory for values of k between 4.4 and 6.1 Nm/rad, approximately, and for k less than 3.7 Nm/rad.

This preliminary results show that the desensitization approach can be used together with the numerical solution of a Two-Point Boundary Value Problem to develop optimal robust trajectories for flexible-link robot described by nonlinear models.

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