Control design for 3D flexible link mechanisms using linearized models

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Abstract. This paper presents an approach to the optimal control of a spatial flexible mechanism. A highly accurate dynamic model of the system is briefly resumed. Then, in order to be able to employ the classical optimal control theory, a linearization of the model with consideration of gravity force is done. After that the chosen optimal control is described, and the most important results of the simulation are presented and discussed.

Keywords: Flexible Links Mechanisms, Dynamic Model, Spatial Mechanism, Control

1 Introduction

Robot motion control is an important criterion for robot manufactures, so the current investigations are focused on increasing the robot performance, robot cost reduction, safety improvements, and increasing new functionalities. Therefore, there is a need to continuously improve the mathematical models and control methods in order to achieve conflicting requirements, such as performance increasing of a weight-reduced robot, with lower mechanical stiffness and more involved vibration modes.

Vibration control of flexible mechanisms as a subset of robot motion control is still an open issue in scientific researches (Tokhi and Azad 2008). A large amount of work has been carried out in the field of flexible mechanism modeling, analysis, and control since the early 1970s. Several techniques are currently available for modeling flexible mechanisms. Most researchers have concentrated their investigation on the describing of accurate mathematical models both for single body and multi-body system (Benosman, Boyer et al. 2002, Dwivedy and Eberhard 2006, Tokhi and Azad 2008). The classical approaches applied in flexible multi-body systems deals with mechanisms featuring large displacement and small deformations. Two main techniques have been adopted in literature (Naganathan and Soni 1988, Nagarajan and Turcic 1990, Kalra and Sharan 1991, Ge, Lee et al. 1997, Martins, Mohamed et al. 2003, Dwivedy and Eberhard 2006): the Finite Element Method (nodal approach) and the Assumed Mode Method (modal approach). Rigid body and elastic motion coupling effects have been considered in different works and approaches, firstly by considering only the effect of the rigid body motion on the elastic deformation (Naganathan and Soni 1988, Kalra and Sharan 1991) and then by considering also the effect of the elastic deformation on the rigid body motion (Nagarajan and Turcic 1990). Floating Frame of Reference (FFR) formulation (Shabana 1997, Shabana 2005) is the consequence of these works.

In this paper a linearized model is developed with the aim of designing model-based control techniques for spatial flexible link mechanisms. Linear models are often used to develop control strategies for this class of mechanism, including robust control (Caracciolo et.al 2005), model predictive control (Boscariol et.al 2010, Boscariol et. al 2009, Boscariol and Zanotto 2012) and sliding mode control (Kurod and Dixit 2012), just to cite a few notable works on the subject. The accuracy of the linearization procedure introduced in this paper is measured by a comparison with the nonlinear model, and its use is demonstrated trough the development of a LQR position and vibration control.

2 Dynamic Model of a Flexible Mechanism

One of the most studied problems in flexible robotics is dynamic modeling. Differently to conventional rigid robots, the elastic behavior of flexible robots makes the mathematical deduction of the models, which govern the real physical behavior, quite difficult. Here the method used for accurate modeling of the systems with large displacements and small elastic deformation is based on the Equivalent Rigid Link System (ERLS) concept which first was introduced for a planar mechanisms (Giovagnoni 1994), and then expanded to spatial environment in (Vidoni, Gasparetto et. al 2012; Vidoni, Gasparetto et al. 2013) which is briefly recalled in this section. According to the work (Boscariol, Gasparetto et. al 2013), the ERLS-FEM dynamic model for flexible-link mechanisms is described by the ODE system of equations:

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{M}J \\ \boldsymbol{J}^T\boldsymbol{M} & \boldsymbol{J}^T\boldsymbol{M}\boldsymbol{J} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{q}} \end{bmatrix} =$$
$$\begin{bmatrix} -2(\boldsymbol{M}_{G1}+\boldsymbol{M}_{G2})-\alpha\boldsymbol{M}-\beta\boldsymbol{K} & -\boldsymbol{M}\dot{\boldsymbol{J}} & -(\boldsymbol{M}_{C1}+2\boldsymbol{M}_{C2}+\boldsymbol{M}_{C3})-\boldsymbol{K} \\ \boldsymbol{J}^T(-2(\boldsymbol{M}_{G1}+\boldsymbol{M}_{G2})-\alpha\boldsymbol{M}) & -\boldsymbol{J}^T\boldsymbol{M}\boldsymbol{J} & -\boldsymbol{J}^T(\boldsymbol{M}_{C1}+2\boldsymbol{M}_{C2}+\boldsymbol{M}_{C3}) \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{u}} \\ \dot{\boldsymbol{q}} \\ \boldsymbol{q} \end{bmatrix} +$$
$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{I} \\ \boldsymbol{J}^T\boldsymbol{M} & \boldsymbol{J}^T \end{bmatrix} \begin{bmatrix} \mathbf{g} \\ \boldsymbol{f} \end{bmatrix} \tag{1}$$

in which $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{J}$ is the Jacobian matrix of the manipulator, $\boldsymbol{M}_{G1}$ and $\boldsymbol{M}_{G2}$ are the Coriolis contribution terms, $\boldsymbol{K}$ is the stiffness matrix, and $\boldsymbol{M}_{C1}$, $\boldsymbol{M}_{C2}$ and $\boldsymbol{M}_{C3}$ are the terms introduced by the centrifugal terms of acceleration. $\boldsymbol{f}$ is the vector of nodal forces, $\boldsymbol{g}$ is the gravity vector, $\boldsymbol{q}$ is the vector of rigid displacements and $\boldsymbol{u}$ is the vector of nodal displacements. Rayleigh damping has been considered in the model, through α and β constants.

3 Linearized Model

The dynamic model represented by eq. (1) is nonlinear, due to the quadratic relation between the nodal acceleration and the velocities of the free coordinates. Thus it cannot be used to design a linear-model based control. In order to develop a state-space form linearized version of the dynamic system of eq. (1) a linearization procedure has been developed. First of all, eq. (1) can be written in the following form, by defining a state vector $\boldsymbol{x}(t)$ and an input vector $\mathbf{v}(t)$:

$$\boldsymbol{A}(\boldsymbol{x}(t))\dot{\boldsymbol{x}}(t) = \boldsymbol{B}\big(\boldsymbol{x}(t)\big)\boldsymbol{x}(t) + \boldsymbol{C}(\boldsymbol{x}(t))\boldsymbol{v}(t)$$

In which matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ do not depend on $\boldsymbol{v}(t)$. If $\boldsymbol{x_e}$ is a steady equilibrium point for the system in eq. (1), a linearization procedure can be set by applying a Taylor series expansion:

$$\boldsymbol{A}\big(\boldsymbol{x}_e + \Delta\boldsymbol{x}(t)\big)\big(\dot{\boldsymbol{x}}_e + \Delta\dot{\boldsymbol{x}}(t)\big) = \boldsymbol{B}\big(\boldsymbol{x}_e + \Delta\boldsymbol{x}(t)\big)\big(\boldsymbol{x}_e + \Delta\boldsymbol{x}(t)\big) + \boldsymbol{C}\big(\boldsymbol{x}_e + \Delta\boldsymbol{x}(t)\big)\big(\boldsymbol{v}_e + \Delta\boldsymbol{v}(t)\big) \tag{2}$$

Since $\boldsymbol{x_e}(t)$ is an equilibrium point for the system, the following equation holds:

$$\boldsymbol{B}(\boldsymbol{x}_e)\boldsymbol{x_e} + \boldsymbol{C}_e(\boldsymbol{x}_e)\boldsymbol{v_e} = \boldsymbol{A}(\boldsymbol{x}_e)\dot{\boldsymbol{x}}_e = 0 \tag{3}$$

Therefore the system linearized around the equilibrium point can be written as:

$$\boldsymbol{A}(\boldsymbol{x}_e)\Delta\dot{\boldsymbol{x}}(t) = \left[\boldsymbol{B}(\boldsymbol{x}_e) + \left(\frac{\partial \boldsymbol{B}}{\partial x}\bigg|_{\boldsymbol{x}=\boldsymbol{x}_e} \times \boldsymbol{x}_e\right) + \left(\frac{\partial \boldsymbol{C}}{\partial x}\bigg|_{\boldsymbol{x}=\boldsymbol{x}_e} \times \boldsymbol{v}_e\right)\right]\Delta\boldsymbol{x}(t) + \boldsymbol{C}(\boldsymbol{x}_e)\Delta\boldsymbol{v}(t) \tag{4}$$

The matrices in eq. (4) are constant, so we have obtained a linear model in the form:

$$\boldsymbol{A}\Delta\dot{\boldsymbol{x}}(t) = \boldsymbol{B}\Delta\boldsymbol{x}(t) + \boldsymbol{C}\Delta\boldsymbol{v}(t) \tag{5}$$

The constant matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ can be evaluated as:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{M} & \boldsymbol{MJ} & 0 & 0 \\ \boldsymbol{J}^T\boldsymbol{M} & \boldsymbol{J}^T\boldsymbol{MJ} & 0 & 0 \\ 0 & 0 & \boldsymbol{I} & 0 \\ 0 & 0 & 0 & \boldsymbol{I} \end{bmatrix}_{\boldsymbol{x}=\boldsymbol{x}_e} \tag{6}$$

$$\boldsymbol{B} = \begin{bmatrix} -2\boldsymbol{M}_G - \alpha\boldsymbol{M} - \beta\boldsymbol{K} & 0 & -\boldsymbol{K} & \boldsymbol{B}_{14} \\ \boldsymbol{J}^T(-2\boldsymbol{M}_G - \alpha\boldsymbol{M} - \beta\boldsymbol{K}) & 0 & 0 & \boldsymbol{B}_{24} \\ \boldsymbol{I} & 0 & 0 & 0 \\ 0 & \boldsymbol{I} & 0 & 0 \end{bmatrix}_{\boldsymbol{x}=\boldsymbol{x}_e} \tag{7}$$

in which:

$$\boldsymbol{B}_{14} = -\frac{\partial \boldsymbol{K}}{\partial \boldsymbol{q}} \times \boldsymbol{u}_e + \frac{\partial \boldsymbol{F}_g}{\partial \boldsymbol{q}} \tag{8}$$

$$\boldsymbol{B}_{24} = -\frac{\partial \boldsymbol{J}^T}{\partial \boldsymbol{q}} \times \boldsymbol{F}_g + \boldsymbol{J}^T \times \frac{\partial \boldsymbol{F}_g}{\partial \boldsymbol{q}} \tag{9}$$

Matrix $\boldsymbol{C}$ remains unchanged after the linearization process, since it is composed of only zeros and ones. Eq. (5) can be brought to the most common form of a Linear Time Invariant (LTI) model by using the simple relations $\boldsymbol{F}_{lin} = \boldsymbol{A}^{-1}\boldsymbol{B}$ and $\boldsymbol{G}_{lin} = \boldsymbol{A}^{-1}\boldsymbol{C}$:

$$\begin{cases} \Delta\dot{\boldsymbol{x}}(t) = \boldsymbol{F}_{lin}\Delta\boldsymbol{x}(t) + \boldsymbol{G}_{lin}\boldsymbol{v}(t) \\ \boldsymbol{y}(t) = \boldsymbol{H}_{lin}\boldsymbol{x}(t) + \boldsymbol{D}_{lin}\boldsymbol{v}(\boldsymbol{t}) \end{cases} \tag{10}$$

4 Reference Mechanism

The mechanism chosen as the basis of the simulations is a L-shape mechanism, made by two steel rods, connected by a rigid aluminum joint (figure 1). The rotational motion of the first link, which is rigidly connected to the motor, can be imposed through a torque-controlled actuator. The whole mechanism can swing in 3D environment, so the effects of gravity on both the rigid and elastic motion of the mechanism can be considered and taken into account in the formulation.

The mechanism shown in figure 1 is made by two aluminum beam whose length is 0.5 m, and their square section is 8 mm wide. Two Euler-Bernoulli finite elements have been used for each link (figure 1). Since each finite element has 12 degrees of freedom, every link has 18 degrees of freedom. After assembling the 2 links and considering the constraints fixed by the kinematic couplings and neglecting one of the nodal displacements in order to make the system solvable (see (Vidoni, Gasparetto et al. 2013)), the resulting system is described by 24 nodal elastic displacements and one rigid degree of freedom, as shown in figure 1.

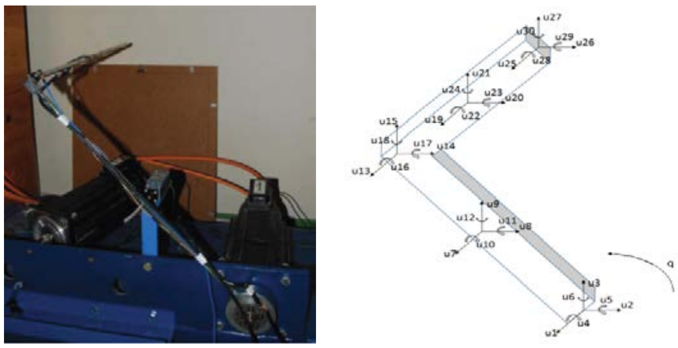

Fig. 1 The mechanism built in the laboratory for the experimental validation of the model (left), FEM discretization and nodal displacements (right)

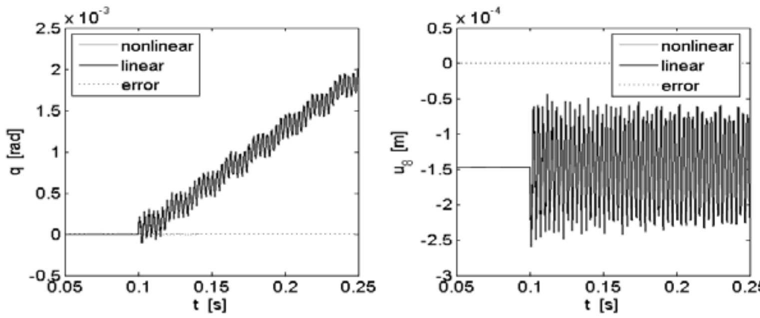

Fig. 2 Impulsive response: comparison between nonlinear and liner model: angular position q (left), nodal displacement u_8 (right)

4.1 Accuracy of the Linearized Model

In order to estimate the accuracy of the linearized model, a comparison between the impulsive response for linear and nonlinear models is set. The mechanisms is fed with a 5 *Nm* torque impulse applied to the crank with 0.1 *sec* delay. The initial configuration has been arbitrarily chosen as $q_{eq} = 0$ deg, but similar results can be obtained for any choice of the linearization configuration of the mechanism. The comparison is set in terms of rigid displacement q and of nodal displacement $u_{8,}$ but it could be extended also to all the other nodal displacements belonging to the model leading to similar results. As it can be seen from figure 2 the linearized model shows a very high level of accuracy, both in terms of q displacement and nodal displacement u_8. It can be noticed that the impulsive response of the two models lead to two very similar responses, since the error, defined as the difference between the two responses, is almost negligible.

5 Results

The linearized dynamic model of the system, obtained in Section 4 eq. (10), can be used to synthesize an optimal LQR controller. The output vector $\boldsymbol{y}$ was defined to be the full state vector (i.e. $\boldsymbol{H}$ was taken as the identify matrix).
The goal is to determine the control action $\tau(t)$, which allows minimizing the performance index $\boldsymbol{W}$**,** defined as:

$$\boldsymbol{W} = \int_0^\infty [\boldsymbol{y}^T(t)\boldsymbol{Q}\boldsymbol{y}(t) + \tau^T(t)\boldsymbol{L}\tau(t)]dt = \int_0^\infty [\boldsymbol{x}^T(t)\boldsymbol{H}^T\boldsymbol{Q}\boldsymbol{H}\boldsymbol{x}(t) + \tau^T(t)\boldsymbol{L}\tau(t)]dt \tag{12}$$

$\boldsymbol{Q}$ and $\boldsymbol{L}$ matrices are used to tune the control system, by defining the weight of each value of the state vector $\boldsymbol{x}$(t) and of control action $\tau(t)$ on the cost function c. The resulting gain matrix $\boldsymbol{K}$ can be evaluated using the well-known results of optimal control theory (Kirk 2012).

The results of two numerical tests are reported in figure 3 and 4. The results refer to two test case: in the first one only the angular position q is weighted in matrix Q, while in the second one also the elastic displacements are weighted. The initial position of the L-shape mechanisms is taken as $q = 90$ deg. A step reference input with amplitude $\Delta q = 4$ deg with *0.05 s* delay was given to the mechanisms actuator which is an electrical motor. Our aim is to reach the defined position for the rigid DOF coordinate with a limited amplitude of vibrations.

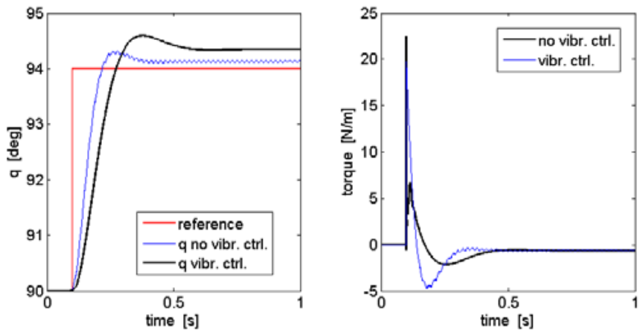

Fig. 3. LQR control: angular position (left), applied torque (right), with and without vibration control

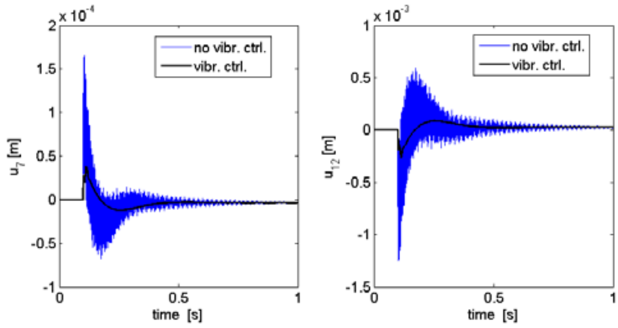

Fig. 4. LQR control: time evolution of elastic displacements u_7 and u_{12}, with and without vibration control

Figure 3 shows the step response of the free coordinate (left) and the torque produced by the actuator (right). The amplitude of nodal vibrations u_7 and u_{12} are shown in figure 4. All the simulations have been run with the linear controller acting on the nonlinear mechanism. It can be seen that the LQR control can achieve a good vibration damping: the amplitude of nodal displacement are kept constant within 0.5 s approximately. The angular displacement q can track quite well the reference signal, but with a constant error. This could be reduced by introducing an integral action in the controller.

7 Conclusion

In this work a linearized model for spatial flexible link mechanism has been developed. The dynamic model can account for gravity acting on any direction. The accuracy of the linearized model is evaluated trough a comparison with the response of the nonlinear model. The linearization procedure allows to develop model-based control strategies for this class of mechanisms. A model design procedure has been applied to build a LQR position and vibration control. The results show that the synthesized LQR produces fast response with a good vibration damping.

References

Benosman, M., F. Boyer, G. Le Vey and D. Primault (2002). "Flexible Links Manipulators: from Modelling to Control." Journal of Intelligent and Robotic Systems **34**(4): 381-414.

Boscariol, P., A. Gasparetto and V. Zanotto (2009). "Vibration reduction in a flexible link mechanism trough the synthesis of an MPC controller" Mechatronics, 2009. ICM 2009. IEEE International Conference on. IEEE, 2009.

Boscariol, P., A. Gasparetto and V. Zanotto (2010). "Active position and vibration control of a flexible links mechanisms using model-based predictive control" Journal of dynamic systems, measurement and control **132**(1)

Boscariol, P., Zanotto, V. (2012). "Design of a controller for trajectory tracking for compliant mechanisms with effective vibration suppression". Robotica, **30**(1): 15-29.

Boscariol, P. Gasparetto, A. Giovagnoni, M., Moosavi A.K., Vidoni R. (2013) "On the modeling of flexible-link robots: First experimental validation of an ERLS-FEM dynamic model", Mechatronics (ICM), 2013 IEEE International Conference on, 298-302.

Caracciolo, R., D. Richiedei, A. Trevisani and V. Zanotto (2005). "Robust mixed-norm position and vibration control of flexible link mechanisms." Mechatronics **15**(7): 767-791.

Dwivedy, S. K. and P. Eberhard (2006). "Dynamic analysis of flexible manipulators, a literature review." Mechanism and Machine Theory **41**(7): 749-777.

Ge, S. S., T. H. Lee and G. Zhu (1997). "A nonlinear feedback controller for a single-link flexible manipulator based on a finite element model." Journal of Robotic Systems **14**(3): 165-178.

Giovagnoni, M. (1994). "A Numerical and Experimental Analysis of a Chain of Flexible Bodies." Journal of Dynamic Systems, Measurement, and Control **116**(1): 73-80.

Kalra, P. and A. M. Sharan (1991). "Accurate modelling of flexible manipulators using finite element analysis." Mechanism and Machine Theory **26**(3): 299-313.

Kirk, D. E. (2012). Optimal control theory: an introduction. DoverPublications

Martins, J. M., Z. Mohamed, M. O. Tokhi, J. Sa da Costa and M. A. Botto (2003). "Approaches for dynamic modelling of flexible manipulator systems." Control Theory and Applications, IEE Proceedings - **150**(4): 401-411.

Naganathan, G. and A. H. Soni (1988). "Nonlinear Modeling of Kinematic and Flexibility Effects in Manipulator Design." Journal of Mechanisms, Transmissions and Automation in Design **110**(3): 243-254.

Nagarajan, S. and D. A. Turcic (1990). "Lagrangian Formulation of the Equations of Motion for Elastic Mechanisms With Mutual Dependence Between Rigid Body and Elastic Motions: Part I—Element Level Equations." Journal of Dynamic Systems, Measurement, and Control **112**(2): 203-214.

Shabana, A. (1997). "Flexible Multibody Dynamics: Review of Past and Recent Developments." Multibody System Dynamics **1**(2): 189-222.

Shabana, A. A. (2005). Dynamics of Multibody Systems, Cambridge University Press.

Tokhi, O. and A. K. M. Azad (2008). "Flexible Robot Manipulators: Modelling, Simulation and Control", Institution of Engineering and Technology.

Vidoni, R., A. Gasparetto., M. Giovagnoni. (2012). "A method for modeling of three-dimensional flexible mechanisms based on an equivalent rigid-link system." Journal of Vibration and Control.

Vidoni, R., A. Gasparetto and M. Giovagnoni (2013). "Design and implementation of an ERLS-based 3-D dynamic formulation for flexible-link robots." Robotics and Computer-Integrated Manufacturing **29**(2): 273-282.