# A minimum energy trajectory algorithm for mechatronic systems with regenerative braking 

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#### Abstract

In literature, several different optimal motion criteria have been proposed, in particular for achieving fast motion while retaining adequate smoothness. Recently, the concept of energy efficiency in automation industry and robotics has become a major topic. In this work, the problem of finding the optimum compromise between the energy required for the movement of the robot joints, the jerk content and the time taken to perform the planned trajectory is addressed for a generic mechatronic drive-motor-transmission-load system. With respect to the available literature, the energy-related term has been computed taking also into account the possibility to regenerate the braking energy, thus splitting the acceleration and deceleration phases.

Numerical results and comparisons show that the proposed approach can potentially bring important energy savings while maintaining a minimal jerk content.


## I. Introduction

In the modern industry, robots, manipulators and tool machines are widely used in the production cycle in order to perform fast, repetitive and precise operations, and, hence, enhance the performance and lower the production costs.
Over the last decades, a significant increase of the problems related to the climate change and the depletion of fossil resources is occurring. As a consequence, both the electricity and the crude oil prices have been increasing in many industrial countries. Thus, energy efficiency and sustainability become important targets in all the engineering fields and are also among the main targets of the European Community. To this end, focusing the attention on automated industrial systems, an integrated approach that exploits optimum motion planning, effective controls, state of the art sensors and actuators, and energy saving techniques and technologies, can allow to design, upgrade and enhance the efficiency of mechatronic systems such as robots, tool machines and automatic systems.
To this purpose, the development of high performance trajectory planning algorithms could give an important contribution. Recently, the relation between vibrations and the consequent possibility of premature joint wear and mechanical failures have been investigated demonstrating a performance enhancement when smooth trajectories are planned [1]. An extended review of the problem can be found in [2], [3].
This work, among the different approaches, focuses on the off-line non model-based techniques.

Thus, a novel trajectory planning algorithm for industrial robots is here developed: it founds an optimal trajectory by adjusting the time distance between consecutive via points in order to minimize a cost function of choice; constraints on physical parameters such as velocity, acceleration and jerk can also be specified as inputs of the procedure. Similar approaches can be found in literature focusing on minimumenergy [4], [5], [6], [7], [8], minimum jerk and hybrid approaches [9], [10], [11].
In particular, the planning algorithm here presented has been developed with the aim to balance traveling time, jerk content and energy losses when regenerative brakes are exploited.

## II. Optimization Problem

The optimization strategy here discussed aims at minimizing/reducing the motor Joule losses during the braking phases by studying a proper off-line non-model based algorithm. Such a reduction is obtained searching a compromise between the allowed increment of the braking time and the reduction of the RMS current value in the same phase. Then, the idea is to allow a longer braking time with the aim to reduce the motor braking torque and, thus, the Joule losses on the motor windings. Such a strategy and evaluation have been applied by considering a motor with an AFE (Active Front End) that allows the recovery of the motor braking energy.
The off-line non model-based trajectory planning usually deals with physical quantities such as joint position, velocity and acceleration. Thus, it is necessary to write the current that flows inner the motor windings, variable to minimize, as a function of these quantities.
The optimization strategy is based on the minimization of a cost functional $J(T, \alpha)$ defined by the weighted sum of the integral of the squared value of the end-effector acceleration during the braking phase, and the overall trajectory time $T$. In such a way, a compromise between the traveling time and the Joule losses can be found.

The following optimization problem is formulated:

$$
\begin{array}{ll}
\min _{U \in \mathcal{U}} & J=K_{1} \sum_{i=1}^{N} h_{i}+K_{2} \int_{T} \operatorname{Jerk}^{2}(t) d t+K_{3} \int_{T_{b}} \alpha^{2}(t) d t \\
\text { s.t. } & g\left(h_{i}\right)=0 \\
& f\left(h_{i}\right) \leq 0 \tag{1}
\end{array}
$$

where: $\alpha(\mathrm{t})$ is the end-effector acceleration and $h_{i}$ is the generic time blend between two via-points. In addition, eq. 1 shows also the constraints that have to be fulfilled in order to have a spline with position, velocity and acceleration continuity, and the constraints that the physical quantities of the trajectory have to satisfy.
The considered mechanical system for which the optimization problem is written and applied is a generic joint that allows both a transmission ratio and a rack and pinion system for the conversion of the motion from rotational to linear.

The rack and pinion system is described by the following equations:

$$
\left\{\begin{array}{ll}
\operatorname{Pm} m_{\text {pinion }}(t) & =\omega_{L}(t) T_{L}(t)  \tag{2}\\
\operatorname{Pm} & \text { rack }(t)
\end{array}=f_{A}(t) v_{L}(t)+M r^{2} \frac{d \omega_{L}}{d t}\right.
$$

where the quantities $P m_{\text {pinion }}(t)$ and $P m_{\text {rack }}(t)$ represent the mechanical power requested by the motor shaft and the rack respectively. $\omega_{L}(t)$ and $T_{L}(t)$ are the angular velocity and the torque reflected to the load, i.e. after the transmission ratio. Moreover, $f_{A}(t), v_{L}(t), M, r$, are the friction force on the end-effector, the linear velocity, the mass of the end-effector and pinion radius.

By equating the two mechanical power expressions in eq.2, the overall load torque after the gear ratio $T_{L}$ can be computed. Then, the following holds:

$$
\begin{equation*}
T_{L}(t)=T_{r}+J_{L} \frac{d \omega_{L}}{d t} \tag{3}
\end{equation*}
$$

where $T_{r}=f_{a} r[\mathrm{Nm}]$ is the equivalent resisting torque and $J_{L}=M r^{2}\left[\mathrm{Kgm}^{2}\right]$ is the equivalent moment of inertia, reflected to the motor shaft.
By reflecting the torque $T_{L}$ on the motor shaft, the motor torque $T_{m}$ can be computed from the following system:

$$
\begin{cases}T_{m}(t) & =T_{L}^{\prime}+J_{m}(t) \frac{d \omega_{m}}{d t}  \tag{4}\\ T_{L}^{\prime}(t) \omega_{m}(t) & =\left(J_{L} \frac{d \omega_{L}}{d t}+T_{r}\right) \omega_{L}(t) \\ \omega_{m}(t) & =K_{r} \omega_{L}(t)\end{cases}
$$

with $\omega_{m}(t)$ the motor angular velocity, $\omega_{L}(t)$ the load angular velocity, $J_{m}$ the motor moment of inertia, $J_{L}$ the load moment of inertia.
By solving the system in 4, the following expression can be found:

$$
\begin{align*}
T_{m}(t) & =\frac{T_{r}}{K_{r}}+\frac{\left(J_{L}+J_{m} K_{r}^{2}\right)}{K_{r}} \frac{d \omega_{L}}{d t}  \tag{5}\\
& =T_{r}^{\prime \prime}+J_{e q} \frac{d \omega_{L}}{d t}=T_{r}^{\prime \prime}+J_{e q} \alpha(t)
\end{align*}
$$

where $\alpha(t)=\frac{d \omega_{L}}{d t}$, is the joint acceleration.
The relation between $\alpha(t)$ and the current on the motor windings $i_{m}$ can be computed from the following system of equations:

$$
\left\{\begin{array}{l}
T_{m}(t)=T_{r}^{\prime \prime}+J_{e q} \alpha(t)  \tag{6}\\
T_{m}(t)=K_{e m} i_{m}
\end{array}\right.
$$

where $K_{e m}$ the electromagnetic constant. Thus, the $i_{m}$ current written as a function of the joint acceleration $\alpha$ results:

$$
\begin{equation*}
i_{m}(t)=\frac{T_{r}^{\prime \prime}+J_{e q} \alpha(t)}{K_{e m} K_{r}} \tag{7}
\end{equation*}
$$

The Joule losses along the braking phase can be estimated as:

$$
\begin{align*}
E_{J} & =R \int_{T_{b}} i_{m}^{2}(t) d t \\
& =\frac{R}{K_{e m} K_{r}} \int_{T_{b}}\left[T_{r}^{\prime \prime 2}+2 J_{e q} T_{r}^{\prime \prime} \alpha(t)+J_{e q}^{2} \alpha^{2}(t)\right] d t \tag{8}
\end{align*}
$$

By looking at the Joule losses expression $E_{J}$, there is a term related to the integral value of $\alpha^{2}(t)$, as well as in the cost functional $J(T, \alpha)$ in eq.1. This justifies the role of the minimization of the integral of the squared value of the acceleration in the trajectory planning problem under study. Such a minimization is made only during the braking phase $\left(T_{b}\right)$ and not during the acceleration time in order to allow the system to exploit the overall motor nominal torque.

## III. Cost function

Now, given the cost functional in eq. 1 the target is to minimize it under the problem constraints.
The first term does not need any manipulation to be implemented and computed while the second term has to be manipulated and expressed in a suitable manner. Since the chosen primitives are cubic splines, the acceleration is piecewise continuous and the jerk is piecewise constant. Thus, the integral can be written trough the following sum:

$$
\begin{equation*}
\int_{T} \operatorname{Jerk}^{2}(t)=\sum_{i=1}^{N+1}\left(\frac{\alpha_{i+1}-\alpha_{i}}{h_{i}}\right)^{2} h_{i} \tag{9}
\end{equation*}
$$

in which $i$ is the polynomial index.
By recalling the mathematical expression of the spline as a polynomial function [2], the term can be rewritten as:

$$
\begin{align*}
\int_{T} \text { Jerk }^{2}(t) & =\sum_{i=1}^{N+1}\left(\frac{6 a_{3}^{i} h_{i}+2 a_{2}^{i}-2 a_{2}^{i}}{h_{i}}\right)^{2} h_{i}  \tag{10}\\
& =\sum_{i=1}^{N+1}\left(6 a_{3}^{i}\right)^{2} h_{i}
\end{align*}
$$

where $a_{3}^{i}$ is the 4-th coefficient of the i-th polynomial.
The third term in the cost function (1) is proportional to the integral of the squared value of the acceleration during the time blends in which velocity and acceleration differ in sign, i.e. during a braking phase. Again, a sum is used for the numerical evaluation of the integral. In this case the sum represents an approximation of the integral since the acceleration is not piecewise constant in the $h_{i}$. In order to reduce the computational error, the $h_{i}$ is split in small time intervals of $s t e p_{L}$ time amplitude.

The chosen algorithm for the sum calculus results:

$$
\begin{equation*}
\int_{T_{b}} \alpha^{2}(t) d t \simeq \sum_{i=1}^{N+1} \sum_{k=1}^{N_{\text {step }}}\left(6 a_{3}^{i} h_{i k}+2 a_{2}^{i}\right)^{2} \operatorname{Mask}(i, k) \operatorname{step}_{L} \tag{11}
\end{equation*}
$$

where $\operatorname{Mask}(i, k)$ is:

$$
\begin{cases}\text { acceleration }(i, k) & =6 a_{3}^{i} h_{i k}+2 a_{2}^{i}  \tag{12}\\ \operatorname{speed}(i, k) & =3 a_{3}^{i} h_{i k}^{2}+2 a_{2}^{i} h_{i k}+a_{1}^{i} \\ \operatorname{Pwr}(i, k) & =\text { speed } \cdot \text { acceleration } \\ \operatorname{Mask}(i, k) & =0.5 \cdot \frac{P w r-a b s(P w r)}{P w r+e p s}\end{cases}
$$

The $i$ and $k$ indexes refer to the generic $i$-th time interval between two consecutive via-points $h_{i}$ and the generic $k$ th discretization interval inner the $h_{i}$ respectively. To allow a good compromise between the algorithm speed and the integral calculus accuracy, the step $_{L}$ has been chosen equal to 0.01 s . By looking at the $\operatorname{Mask}(i, k)$ term, there is a cost function increment, i.e. it is equal to one, only when velocity and acceleration are discordant.

The small quantity eps that appears inside the mask definition is inserted to avoid the divergence of the ratio when $P w r$ approaches zero.

## A. Constraints

1) Equality Constraints: The equality constraints that must be satisfied by the trajectory are the equations that impose both the traveling along the via-points and the continuity in position, velocity, acceleration and jerk. These requirements are fulfilled by using a classical third order polynomial spline with the introduction of two extra virtual via-points put between the first and the last via-point.

The spline mathematical expression between the via-points is:

$$
\begin{align*}
& S(t)= \\
& a_{3}^{1}\left(t-t_{0}\right)^{3}+a_{2}^{1}\left(t-t_{0}\right)^{2}+a_{1}^{1}\left(t-t_{0}\right)+a_{0}^{1}, \\
& t \in\left[t_{0}, t_{1}\right] \\
& a_{3}^{2}\left(t-t_{1}\right)^{3}+a_{2}^{2}\left(t-t_{1}\right)^{2}+a_{1}^{2}\left(t-t_{1}\right)+a_{0}^{2}, \\
& t \in\left[t_{1}, t_{2}\right] \\
& \cdots  \tag{13}\\
& a_{3}^{N+2}\left(t-t_{N+1}\right)^{3}+a_{2}^{N+2}\left(t-t_{N+1}\right)^{2}+a_{1}^{N+2}\left(t-t_{N+1}\right)+a_{0}^{N+2}, \\
& t \in\left[t_{N+1}, t_{N+2}\right]
\end{align*}
$$

where $t_{k}$ represents the traveling time on the via-point, i.e. problem unknowns. The formulation can be rewritten by means of the following substitution:

$$
\begin{equation*}
\left(t-t_{i}\right) \longrightarrow t_{i}^{\prime}, \quad t_{i}^{\prime} \in\left[0, h_{i}\right] \tag{14}
\end{equation*}
$$

where $i$ is the generic polynomial index.
The constraints, the via-point passage and continuity up to the $2^{\text {nd }}$ order can then be written with the following equation
system:

$$
\left\{\begin{array}{l}
\Pi_{1}(0)=q_{1} \\
\Pi_{1}^{\prime}(0)=0 \\
\Pi_{1}^{\prime \prime}(0)=0 \\
\\
\Pi_{i}(0)=q_{i} \quad \text { with }: \quad i \in[3, \cdots, N]  \tag{15}\\
\Pi_{i}\left(h_{i}\right)=\Pi_{i+1}(0) \quad \\
\Pi_{i}^{\prime}\left(h_{i}\right)=\Pi_{i+1}^{\prime}(0) \\
\Pi_{i}^{\prime \prime}\left(h_{i}\right)=\Pi_{i+1}^{\prime \prime}(0) \\
\\
\Pi_{i}\left(h_{i}\right)=\Pi_{i+1}(0) \quad \text { with }: \quad i \in\{2, N+1\} \\
\Pi_{i}^{\prime}\left(h_{i}\right)=\Pi_{i+1}^{\prime}(0) \\
\Pi_{i}^{\prime \prime}\left(h_{i}\right)=\Pi_{i+1}^{\prime \prime}(0) \\
\\
\Pi_{N+2}(0)=q_{N+2} \\
\Pi_{N+2}^{\prime}(0)=0 \\
\Pi_{N+2}^{\prime \prime}(0)=0
\end{array}\right.
$$

where the i-th polynomial and its time derivatives are defined as:

$$
\begin{align*}
\Pi_{i} & =a_{3}^{i}\left(t_{i}^{\prime}\right)^{3}+a_{2}^{i}\left(t_{i}^{\prime}\right)^{2}+a_{1}^{i}\left(t_{i}^{\prime}\right)+a_{0}^{i}, t_{i}^{\prime} \in\left[0, h_{i}\right](16)  \tag{16}\\
\Pi_{i}^{\prime} & =3 a_{3}^{i}\left(t_{i}^{\prime}\right)^{2}+2 a_{2}^{i}\left(t_{i}^{\prime}\right)+a_{1}^{i}  \tag{17}\\
\Pi_{i}^{\prime \prime} & =6 a_{3}^{i}\left(t_{i}^{\prime}\right)+2 a_{2}^{i}  \tag{18}\\
\Pi_{i}^{\prime \prime \prime} & =6 a_{3}^{i} \tag{19}
\end{align*}
$$

Thus, the system variables are the polynomial coefficients $a_{i}^{j}$ and the time intervals $h_{i}$.
2) Inequality Constraints: These constraints are necessary to limit velocity, acceleration and jerk peak values during the planning phase. Moreover, constraints related to the optimization variables, i.e. the $h_{i}$ widths, are given. They result in the following equations systems:

$$
\begin{align*}
& \left\{\begin{array}{l}
a_{3}^{j} \leq \text { Jerk }_{\max } / 6 \\
a_{2}^{j} \leq \text { Acceleration }_{\max } / 2 \\
a_{1}^{j} \leq \text { Speed }_{\max }
\end{array}\right.  \tag{20}\\
& \left\{\begin{array}{l}
a_{3}^{j} \geq \text { Jerk }_{\min } / 6 \\
a_{2}^{j} \geq \text { Acceleration }_{\min } / 2 \\
a_{1}^{j} \geq \text { Speed }_{\min }
\end{array}\right. \tag{21}
\end{align*}
$$

The $h_{i}$-s are upper bounded by a user defined value while lower bounded by the ratio between the via-point distance and the maximum allowable velocity.

## IV. NUMERICAL SIMULATION

To evaluate the effectiveness of the proposed approach, three different algorithms have been implemented and compared:

- classical spline: fixed and equal time blends between the cubic spline via-points [12];
- SPL3J algorithm: minimum time-jerk algorithm [10], [13];
- SPL3B algorithm: proposed optimum algorithm.

Three different paths have been chosen in order to quantify the performance enhancement in terms of required energy and vibrational content, which is measured trough jerk. The two optimum algorithms have been considered allowing a free total execution time, i.e. with a possible increment of the $1 \%, 5 \%$, $10 \%, 20 \%, 30 \%$ e $50 \%$ with respect to the time required by the basic algorithm.
The chosen paths are:
TRAJECTORY 1: (Fig. 1)
Path (m): $[0,-0.304,0.557,1.100,1.751,1.65,0.86,0.80,0]$ Time: 5 s

TRAJECTORY 2: (Fig. 2)
Path (m): [0, 0.4, 0.8, 1.2, 0.8, 0.4, 0]
Time: 5 s
TRAJECTORY 3: (Fig. 3)
Path (m): $[0,0.4,0.8,1.2,0.8,0.4,0,0.4,0.8,1.2,0.8,0.4,0]$ Time: 5 s


Fig. 1. Trajectory 1: obtained with the basic algorithm. Crosses represent the via-points.

Weigths $K_{t}, K_{j}$ and $K_{p w}$ that appear in the cost function (eq. 1) have been properly chosen in order to obtain the proper trajectory times (Tab. I,III,V). In the following Tables, Tab.II, IV, VI, the numerical results related to the implemented trajectories and to the compared algorithms are shown.
The results show, as expected, that the two optimum algorithms outperform the classical spline approach.
Moreover, it is shown that a delay as small as $1 \%$ with respect to the basic trajectory, i.e. the classical third order spline, can bring a noticeable reduction of energy losses. Both algorithms bring energetic improvements that are directly proportional to the allowed delay. By comparing the two optimum approaches in terms of energy expenditure, it can be seen how


Fig. 2. Trajectory 2 obtained with the basic algorithm. Crosses represent the via-points.


Fig. 3. Trajectory 3 obtained with the basic algorithm. Crosses represent the via-points.

|  | SPL3J |  | SPL3B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Delay | $K_{t}$ | $K_{j}$ | $K_{t}$ | $K_{p w}$ | $K_{j}$ |
| $1 \%$ | 42.1 | 0.497 | 233 | 27.05 | 0.598 |
| $5 \%$ | 31.5 | 0.48 | 193.05 | 27.1 | 0.573 |
| $10 \%$ | 10.9 | 0.224 | 229 | 48 | 0.23 |
| $20 \%$ | 30 | 1 | 120 | 26.25 | 1 |
| $30 \%$ | 19 | 1 | 100 | 33.2 | 1 |
| $50 \%$ | 7.87 | 1 | 87.73 | 60.87 | 1 |

TABLE I
Delays and weights for Trajectory 1
the proposed SPL3B algorithm allows to decrease significantly the energy requirement and to obtain better performances with respect to the SPL3J one. Results show an improvement up to

|  | SPL3J |  | SPL3B |  | Enhancement |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Delay | Jerk | Energy | Jerk | Energy |  |
| $1 \%$ | $94.14 \%$ | $70.8 \%$ | $91.15 \%$ | $72.9 \%$ | $7.01 \%$ |
| $5 \%$ | $95.54 \%$ | $74.3 \%$ | $92.24 \%$ | $75.9 \%$ | $6.18 \%$ |
| $10 \%$ | $96.28 \%$ | $77.9 \%$ | $92.28 \%$ | $79.1 \%$ | $5.11 \%$ |
| $20 \%$ | $97.52 \%$ | $82.5 \%$ | $96.48 \%$ | $83.9 \%$ | $7.72 \%$ |
| $30 \%$ | $98.31 \%$ | $86 \%$ | $97.3 \%$ | $87 \%$ | $7.26 \%$ |
| $50 \%$ | $99.2 \%$ | $91 \%$ | $98.2 \%$ | $91.2 \%$ | $2 \%$ |

## TABLE II

Reduction in percentage of the jerk and energy terms of the TWO OPTIMUM ALGORITHM WITH RESPECT TO THE CLASSICAL SPLINE FOR THE TRAJECTORY 1. THE LAST COLUMN GIVES THE ENHANCEMENT in terms of energy saved of the SPL3B with respect to the SPL3B ALGORITHM

|  | SPL3J |  | SPL3B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Dlay | $K_{t}$ | $K_{j}$ | $K_{t}$ | $K_{p w}$ | $K_{j}$ |
| $1 \%$ | 10.45 | 1 | 40 | 23.6 | 1 |
| $5 \%$ | 8.25 | 1 | 35 | 25.4 | 1 |
| $10 \%$ | 6.26 | 1 | 30 | 27.4 | 1 |
| $20 \%$ | 3.73 | 1 | 22 | 30.5 | 1 |
| $30 \%$ | 2.3 | 1 | 17 | 34.5 | 1 |
| $50 \%$ | 0.968 | 1 | 10.6 | 42 | 1 |

TABLE III
Delay and weights for the Trajectory 2

|  | SPL3J |  | SPL3B |  | Enhancement |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Delay | Jerk | Energy | Jerk | Energy |  |
| $1 \%$ | $99.27 \%$ | $34 \%$ | $98.99 \%$ | $38 \%$ | $6.174 \%$ |
| $5 \%$ | $99.4 \%$ | $41.7 \%$ | $99.13 \%$ | $45.5 \%$ | $6.5 \%$ |
| $10 \%$ | $99.53 \%$ | $48.8 \%$ | $99.27 \%$ | $52.3 \%$ | $6.876 \%$ |
| $20 \%$ | $99.7 \%$ | $60.5 \%$ | $99.47 \%$ | $63.1 \%$ | $6.6 \%$ |
| $30 \%$ | $99.79 \%$ | $69 \%$ | $99.6 \%$ | $71 \%$ | $6.48 \%$ |
| $50 \%$ | $99.9 \%$ | $79.8 \%$ | $99.75 \%$ | $80.9 \%$ | $5.203 \%$ |

TABLE IV
Reduction in percentage of the Jerk and energy terms of the TWO OPTIMUM ALGORITHM WITH RESPECT TO THE CLASSICAL SPLINE for the Trajectory 2. The last column gives the enhancement in terms of energy saved of the Spl3B with respect to the SPL3B ALGORITHM.

|  | SPL3J |  | SPL3B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Delay | $K_{t}$ | $K_{j}$ | $K_{t}$ | $K_{p w}$ | $K_{j}$ |
| $1 \%$ | 410 | 1 | 780 | 19 | 1 |
| $5 \%$ | 325 | 1 | 710 | 23.3 | 1 |
| $10 \%$ | 245.5 | 1 | 675 | 31.4 | 1 |
| $20 \%$ | 145.8 | 1 | 500 | 37.1 | 1 |
| $30 \%$ | 90 | 1 | 393 | 43.9 | 1 |
| $50 \%$ | 38.2 | 1 | 276 | 62 | 1 |

TABLE V
Delay and weights for Trajectory 3

|  | SPL3J |  | SPL3B |  | Enhancement |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jerk | Energy | Jerk | Energy |  |
| $1 \%$ | $87.42 \%$ | $40.1 \%$ | $86.94 \%$ | $41.7 \%$ | $3.39 \%$ |
| $5 \%$ | $89.64 \%$ | $47.1 \%$ | $88.74 \%$ | $48.6 \%$ | $2.77 \%$ |
| $10 \%$ | $91.8 \%$ | $53.5 \%$ | $90.67 \%$ | $54.9 \%$ | $3.05 \%$ |
| $20 \%$ | $94.69 \%$ | $64.3 \%$ | $93.6 \%$ | $65.4 \%$ | $3.18 \%$ |
| $30 \%$ | $96.45 \%$ | $71.9 \%$ | $95.42 \%$ | $72.5 \%$ | $2.33 \%$ |
| $50 \%$ | $98.26 \%$ | $81.6 \%$ | $97.28 \%$ | $82.0 \%$ | $1.70 \%$ |

TABLE VI
REDUCTION IN PERCENTAGE OF THE JERK AND ENERGY TERMS OF THE TWO OPTIMUM ALGORITHM WITH RESPECT TO THE CLASSICAL SPLINE for the Trajectory 3. The last column gives the enhancement IN TERMS OF ENERGY SAVED OF THE SPL3B WITH RESPECT TO THE SPL3B ALGORITHM.
$8 \%$ for these simple and not recursive trajetories thus allowing to forecast better performances when more complex paths are to be travelled.

## V. Conclusion

In this work, an optimal trajectory planning technique that takes into account the simultaneous minimization of the trajectory time, the jerk content and the energy losses during the braking/recovery phase has been presented. The algorithm has been implemented in a Matlab simulator and its performances are compared with other two well known planning algorithms showing good performances and the effectiveness of the idea. Future work will cover the experimental validation of the proposed technique.

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