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Energy-Efficient Point-to-Point Trajectory Generation for Industrial Robotic Machines

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ABSTRACT

Robots and mechatronic applications are widely used for process automation in plants and factories. Trajectory planning is a fundamental issue for operating these industrial machines and, in this work, a point-to-point (PTP) trajectory based on a S-curve has been designed to reduce the consumed energy of a typical mechatronic system, i.e. a robotic linear axis made of an electric-motor that moves a payload on a plane by means of a transmission system and a toothed belt. This evaluation is used to find the minimum energy consumption conditions, also taking into account the possibility of using a regenerative brake. The problem is defined and solved for several operative conditions, either in a closed-form or numerically using a genetic algorithm.

Keywords: optimum trajectory planning, energy efficiency, industrial robotic machines, point-to-point, energy recovery.

1 INTRODUCTION

Machine tools, cranes, material handling, robots and, more in general, mechatronic applications are widely used for process automation in plants and factories. Trajectory planning is a fundamental issue for controlling these industrial machines [1].

In the past, the focus has been set mainly on the solution of minimum time problems, i.e. the main objective has been to find the solution that leads to a lower execution time of a task. Later the attention has been cast also to the problem of jerk minimization, which is aimed at the reduction of the vibration phenomena and of the mechanical failure risks [2, 3, 4, 5, 6].

Recently, in the wake of the increasing of the prices of energy and the popularity of energy-saving strategy, the concept of energy efficiency has spread also to industrial applications. In this perspective, the development of energy efficient trajectories is a very promising technique, since it can be effectively applied also to systems that do not include any regenerative braking system, i.e. to the majority of the equipment currently in use. Moreover, more recent technological improvements such as variable frequency devices and energy recovery systems are setting new paths for energetic reduction of industrial applications.

A fundamental distinction, among the available methods adopted for reducing energy consumption by an optimum trajectory planning approach, is the exploitation of a model-based, e.g. [7, 8], or of a model-free approach, e.g. [5]. Model-free approaches are appealing for their generality and their high portability on several different industrial applications. On the other hand, model-based approaches can achieve better results in specific cases. Luckily, many mechatronic industrial systems can be adequately modeled by mathematic models. In [9], an energy saving controller with a model based approach through a typical dynamics of feed drive systems including an inertia term, a viscous friction term, a Coulomb friction term and a back-EMF term is introduced. In [10], a model-based method for reducing the total energy consumption of pick-and-place manipulators for given TCP position profiles by a time-scaling approach is presented. In [11], the simultaneous evaluation of both the energy efficiency and the smoothness in the most significant off-line

non-model based methods and algorithms currently adopted in industrial applications is presented. Focusing on the most recent works, the minimum energy trajectory optimization is treated considering the electrical energy exchange via the shared inverter DC link, thus allowing to find a different energy minimum with respect to the available literature approaches [12, 13].

In this work, the design of a point-to-point (PTP) trajectory, known as the S-curve trajectory, to reduce the consumed energy of a typical mechatronic system, i.e. a robotic linear axis made of an electric-motor that moves a payload on a plane by means of a transmission system and a toothed belt, is addressed. The chosen trajectory is a simple motion trajectory (so as to minimize controller and implementation costs), which consists of an acceleration period, a constant velocity period and a deceleration period. The minimum of the required energy is studied, evaluated and found either in a closed form by mathematical methods or numerically through the use of genetic algorithms. In such a way, the S-curve values that ensure the best results in terms of energy savings are identified under different working conditions, e.g. gravity.

The paper is organized as follows: the energy saving trajectory design is presented in Section 2; Section 3 reports the modeling of the robotic linear axis while in Section 4 the minimum absolute-energy problem is formulated by minimizing the energy consumption.

2 S-CURVE TRAJECTORY

The S-curve trajectory is widely used in industrial applications because it is simply implementable in old and new controllers and assures continuous accelerations and smooth jerk profiles [14].

The S-curve trajectory is implemented through a cycloid motion during the acceleration and deceleration periods that permit to have a limited acceleration value at the start and end of the motion. It is basically described by four parameters: acceleration time, constant velocity time, deceleration time and constant velocity magnitude. In eq.s (1),(2) the S-curve trajectory is described in terms of velocity and acceleration.

$$v(t) = \begin{cases} v_o/2(1-\cos(\omega_1 t)) & \text{if } t \in [0,t_1), \\ v_o & \text{if } t \in [t_1, t_1+t_2), \\ v_o/2(1+\cos(\omega_3 t')) & \text{if } t \in [t_1+t_2, t_1+t_2+t_3), \end{cases} \tag{1}$$

$$a(t) = \begin{cases} v_o/2(\omega_1 \sin(\omega_1 t)) & \text{if } t \in [0,t_1), \\ 0 & \text{if } t \in [t_1, t_1+t_2), \\ v_o/2(\omega_3 \sin(\omega_3 t')) & \text{if } t \in [t_1+t_2, t_1+t_2+t_3), \end{cases} \tag{2}$$

where t_1, t_2, t_3 and v_o are the acceleration time, the constant velocity time, the deceleration time and the maximum velocity, respectively. $t' = t - t_1 - t_2$, $\omega_1 = \pi/t_1$, $\omega_3 = \pi/t_3$.

If $t_1 \neq t_3$, it is possible to define the following relations:

$$v_0 = 2L/(t_1 + 2t_2 + t_3), \quad t_2 = T - t_1 - t_3$$

Fig. 1 on the following page shows the S-curve position, velocity and acceleration profiles for $t_1 = t_3 = 0.5s$ and the total motion time $T = 3s$.

3 ELECTRO-MECHANICAL MODEL OF A ROBOTIC LINEAR AXIS AND ENERGY FORMULATION

The system of choice is composed on an electric motor that moves a payload, e.g. a cartesian robot axis, in a horizontal or vertical plane by means of a toothed belt with a reduction ratio equal to K_r (Fig. 2 on the next page). The model takes into account the load inertia, the viscous and Coulomb friction, as well as the resistive loss in the motor windings. According to the S-curve trajectory, the motor torque necessary to move the payload is described by the equation (3):

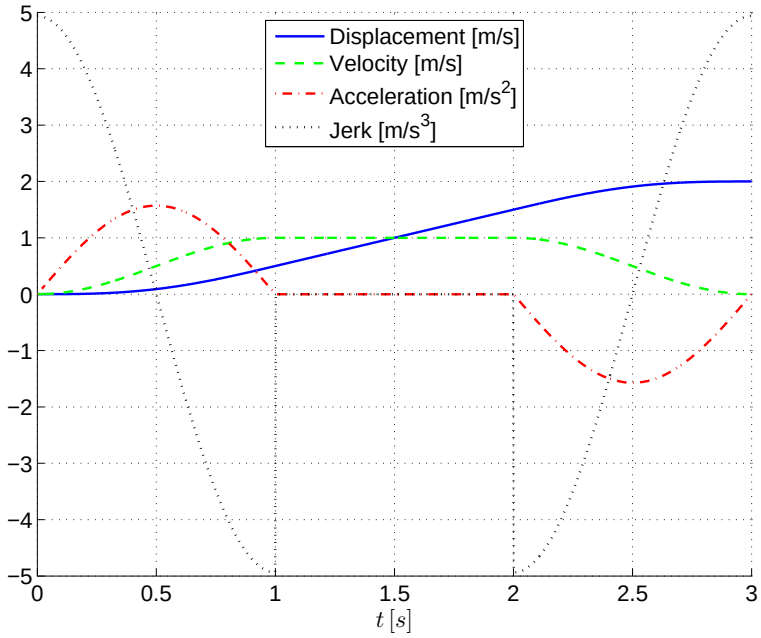


Figure 1: Position, velocity, acceleration and jerk for $t_1 = t_3 = 1s$.

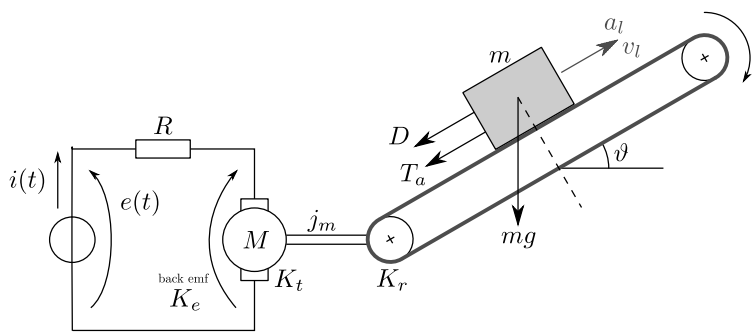


Figure 2: Model of the mechatronic system under evaluation.

$$\tau(t) = \left[\frac{j_m}{K_r} + mK_r\right] a_l(t) + [DK_r]\, v_l(t) + [mgK_r \sin\vartheta + T_a K_r] \tag{3}$$

where j_m is the motor moment of inertia, m the load (l) mass, D the viscous friction coefficient, ϑ the inclination angle and T_a the dynamic friction.

Therefore, the instantaneous current and voltage in the motor phase are:

$$i(t) = \frac{1}{K_t}\left\{\left[\frac{j_m}{K_r} + mK_r\right] a_l(t) + [DK_r]\, v_l(t) + [mgK_r \sin\vartheta + T_a K_r]\right\} \tag{4}$$

$$e(t) = Ri(t) + \frac{K_e}{K_r} v_l(t) \tag{5}$$

with K_t the torque constant and K_e the back-EMF constant.

Then, the instantaneous power can be expressed as:

$$P(t) = Ri(t)^2 + \frac{K_e}{K_r} v_l(t) i(t) \tag{6}$$

Eq. 6 is composed of two terms: the first one is the power loss in the motor winding while the second one is the power used to move the payload. If the latter is positive, the system is in direct motion (i.e. the motor is providing energy to the load), otherwise retrograde motion occurs. In the latter situation the drive system can recover energy since the motor is actually working as a brake (i.e. it is providing a negative work). Starting from the power formulation in eq. 6, it is possible to determine the overall instantaneous energy as:

$$E = E_{res} + E_{m1} + E_{m2}$$

where E_{res} is the energy loss due to the motor windings, E_{m1} the energy consumed to move the load ($E_{m1} > 0$) and E_{m2} the energy recovered when the motor acts as a generator ($E_{m2} < 0$).

A simpler method to calculate the total energy, therefore without taking into consideration the three contributions as separated, is to compute the integral:

$$E = \int e(t)i(t)dt$$

Now, following the approach in [9], it is possible to find the energy formulation for the modeled mechanical system in the generic case $t_1 \neq t_3$ as:

$$E = \frac{(\frac{1}{2}\frac{c1\pi^2}{t_1} + \frac{1}{2}\frac{c1\pi^2}{t_3} + 4c2(T - t_1 - t_3) + \frac{3}{2}c2t_1 + \frac{3}{2}c2t_1)L^2}{(2T - t_1 - t_3)^2} + c3L + c4T \tag{7}$$

where:

$$\begin{aligned}
&b1 = (mg + T_a)K_r/K_t, && b2 = DK_r/K_t, && b3 = (j_m/K_r + mK_r)/K_t,\\
&b4 = Rb1, && b5 = Rb2 + K_e/K_r, && b6 = Rb3,\\
&c1 = b3b6, && c2 = b2b5, && c3 = b1b5 + b2b4,\\
&c4 = b1b4, && c5 = b1b6 + b3b4, && c6 = b2b6 + b3b5.
\end{aligned}$$

4 ENERGY CONSUMPTION OPTIMIZATION

Once fixed the motion displacement, the S-curve trajectory is described by three parameters: the acceleration time, the deceleration time and the total motion time.

In the following, the minimum of the energy consumption is addressed and found, either in a closed-form or through a genetic algorithm, acting on the three variables.

Three main cases for the three variables are studied:

- energy optimization with $t_1 = t_3$ free, total time fixed;
- energy optimization with $t_1 = t_3$ and total time free;
- energy optimization with t_1, t_3 and total time free.

4.1 Energy optimization with $t_1 = t_3$ free, total time fixed

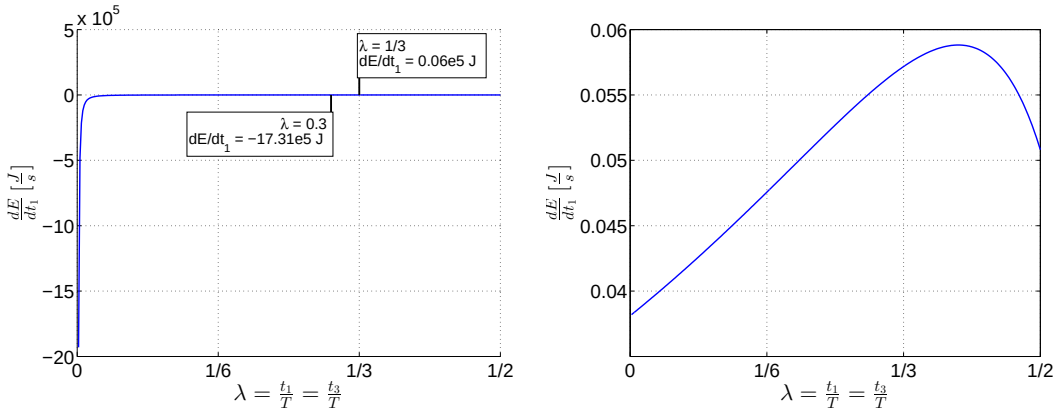
The energy minimization in this first case is the easiest one. Indeed, it is possible to obtain a simple energy formulation, shown in eq. (8), starting from the eq. (7), by forcing $t_1 = t_3$.

$$E = \frac{(-5c2t_1^2 + 4c2Tt_1 + c1\pi^2)L^2}{4t_1(T - t_1)^2} + c3L + c4T \tag{8}$$

The consumed energy depends only on the acceleration time t_1. To find a minimum or a maximum for the equation, the first time-derivative of eq. (8) has to be computed, eq. (9).

Table 1: Test-case mechanical and electrical parameters

mass	m	$10kg$
track length	L	$2m$
transmission ratio	K_r	$0.016m/rad$
torque constant	K_t	$0.14Nm/A$
back-EMF constant	K_e	$0.08Vs/rad$
motor moment of inertia	j_m	$0.00035kgm^2$
electrical motor resistance	R	0.22Ω
gravity acc.	g	$9.81m/s^2$
viscous friction	D	$0.1Ns/m$
dynamic friction	T_a	$30N$
inclination angle	ϑ	0°



(a) *energy first derivative*, $b3 = 10$.

(b) *energy first derivative*, $b3 = 1 \cdot 10^{-7}$.

Figure 3: Different energy first derivative, according to $b3$ value, $T = 3s$.

$$\frac{dE}{dt_1} = \frac{(-5c2t_1^3 + 3c2Tt_1^2 + 3c1\pi^2 t_1 - c1T\pi^2)L^2}{2t_1^2(T-t_1)^3} = 0 \tag{9}$$

It is worthwhile to highlight how the first derivative of the energy is heavily affected by the value of $b3 = (j_m/K_r + m * K_r)/K_t$ that appears in the $c1$ term. In Fig. 3, the first time derivative of the energy is shown for two different $b3$ values; by inspecting the graph and by taking into consideration the definition of $b3$, it can be seen that if the load inertia is relatively smaller than the torque constant, i.e. $b3$ is small, the first time derivative of the energy is always positive. Thus, the energy function decreases with the increase of the acceleration time. If a λ term equal to the ratio between the acceleration time t_1 and the total time T is introduced, the optimum energy value is found by imposing λ, and thus the acceleration time, as small as possible.

In the other case, if $b3$ is big enough, see the plot in Fig. 3(a), the first energy derivative changes both its magnitude and its sign during the acceleration time. Hence, a point of minimum in the energy function can be found at $\lambda = 1/3$. In this work, $b3$ is considered big enough to have a point of minimum for the energy function.

The proposed optimization methods are tested on a benchmark system, whose physical values are shown in Table. 1. To better analyze the energy consumption, attention has to be paid to determine and evaluate the energy consumption both in the total motion displacement and in each of the three parts that compose the S-curve trajectory, i.e. during the acceleration phase, the constant velocity phase and in the deceleration phase.

As shown in Fig. 4, if the λ value is small, the energy consumption during all the three time-blends is high and the energy required during the constant velocity time decreases when λ increases. On the other hand, if the value of λ is high, the energy of the other two blends increases and vice-versa for low λ values. Thus, the sum of the three parts confirms the minimum energy condition with $\lambda = 1/3$.

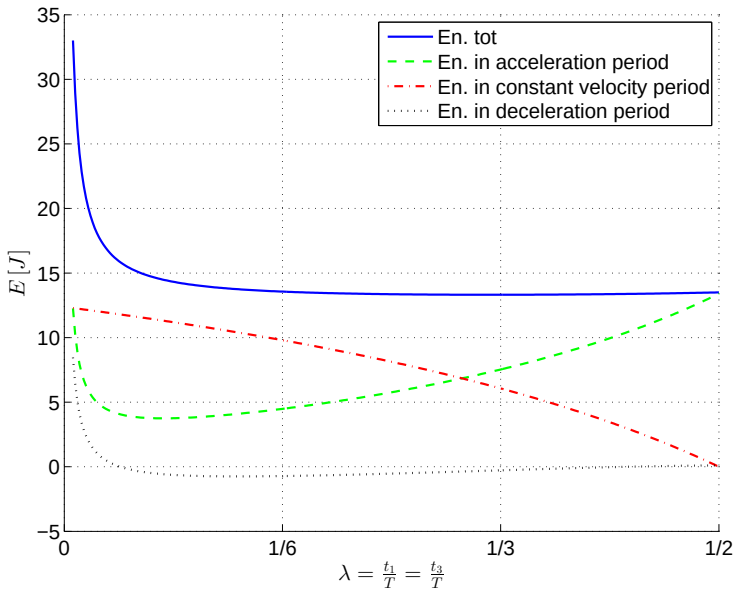


Figure 4: Total energy, energy during acceleration period, energy during constant velocity period, energy during deceleration period, at λ varying, with $t_1 = t_3$, $T = 3s$.

Assuming the displacement in the horizontal plane, i.e. without the effect of the gravity force, it is possible to investigate the energy consumption or the energy recoverable, with respect to the λ value.

In Fig. 5 on the next page, the different energy contributions are shown. For each value of λ, part of the energy is consumed or lost and part is recovered during the load deceleration phase. The main issue in reducing the energy wasted is to reduce the energy used and lost in the armature resistance. Indeed, the global consumption for all the λ values, overlaps the energy dissipations in the resistance.

4.2 Energy optimization with $t_1 = t_3$ and Total time free

The energy minimization in the second case allows to change the acceleration time, the deceleration time and the total time as well. The only binding assumption is that the acceleration time has to be equal to the deceleration time. So, the set of variables to be optimized includes t_1 and T.

Again, the energy formulation to be taken into consideration is given by eq. (7). By computing the points of maximum and minimum:

$$\begin{cases} \frac{dE}{dt_1} = \frac{\left(-\frac{1}{4}\right)L^2\left(-3\,\mathrm{c1}\,\pi^2\,\mathrm{t_1}+T\,\mathrm{c1}\,\pi^2+5\,\mathrm{c2}\,\mathrm{t_1}^3-3\,T\,\mathrm{c2}\,\mathrm{t_1}^2\right)}{\mathrm{t_1}^2\cdot(T-\mathrm{t_1})^3} = 0 \\ \frac{dE}{dT} = \mathrm{c4} - \frac{\frac{\mathrm{c1}\,\pi^2 L^2}{2} - \frac{3\,\mathrm{c2}\,L^2\,\mathrm{t_1}^2}{2} + T\,\mathrm{c2}\,L^2\,\mathrm{t_1}}{\mathrm{t_1}\,(T-\mathrm{t_1})^3} = 0 \end{cases} \tag{10}$$

the Hessian with respect to the variables t_1 and T can be evaluated and the minimum energy solution can be found. By performing all the steps of the method, the point of minimum $\lambda = 0.32$ and $T = 4.07$s is obtained.

The graph in Fig. 6 on the following page highlights the effects of varying both t_1 and T on the total energy consumption. To summarize, in the case that $t_1 = t_3$ and with free total execution time

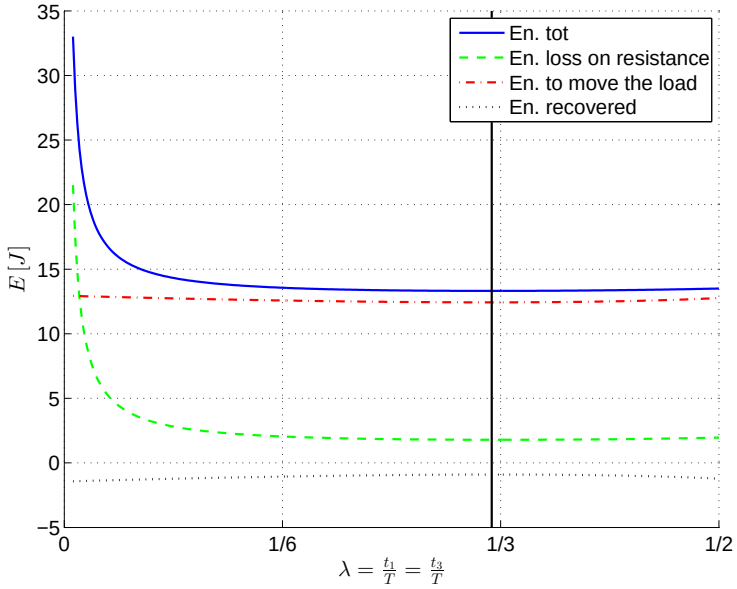


Figure 5: Energy dissipation on the resistance, energy consumed to move the load, energy recovered, with respect to λ. The vertical black line represents the energy minimum value.

T, the energy efficiency is maximized for when the acceleration, the deceleration and the constant velocity phases have equal duration.

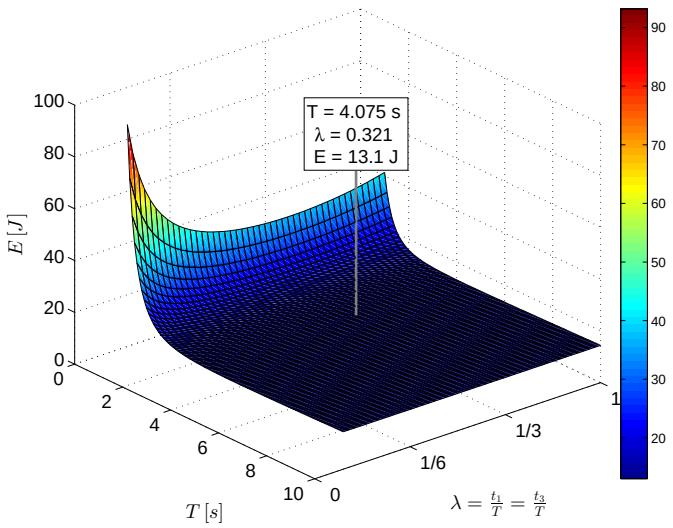


Figure 6: Energy consumption with respect of t_1 for different T slopes.

4.3 Energy optimization with t_1, t_3 e Total time free

The energy minimization in this case takes into account the most general condition, i.e. the one in which t_1, t_3 e T are free.

The problem has to deal with the optimization of an equation in three variables. The optimization problem is in this case solved numerically, through a genetic algorithm procedure. The genetic algorithm (GA) optimization belongs to the larger class of evolutionary algorithms where a population of candidate solutions to an optimization problem is evolved toward better solutions. Here, the "ga" MATLAB function has been exploited for finding a local minimum of an object function *fitnessfcn* under linear bounds on t_1, t_3 and T. The solution of the optimization problem, i.e. the minimum overall power consumption, is achieved for: $\lambda_1 = 0.31$, $\lambda_3 = 0.34$ and $T = 4.07$s.

In this case, the optimum total motion time T is higher than the previously fixed one, i.e. $3s$, demonstrating how it is possible to reduce the energy consumption with the three variables optimization. However, as in the previous cases, the optimum values for the acceleration, deceleration and constant velocity phases are very near to the $1/3$ condition allowing to assess that the equal distribution of the phase along the whole motion time allows to obtain at least a quasi minimum energy consumption.

4.3.1 Gravity effect

If the slope angle ϑ changes from a flat condition up to a vertical condition, the energy E increases. This can be seen looking at the current eq. (4), as shown in Fig. 7. Since the gravity effect does not depend on the total time T, the acceleration t_1 and deceleration t_3 time but only on the configuration, i.e. the slope angle ϑ, the minimum energy configuration does not change.

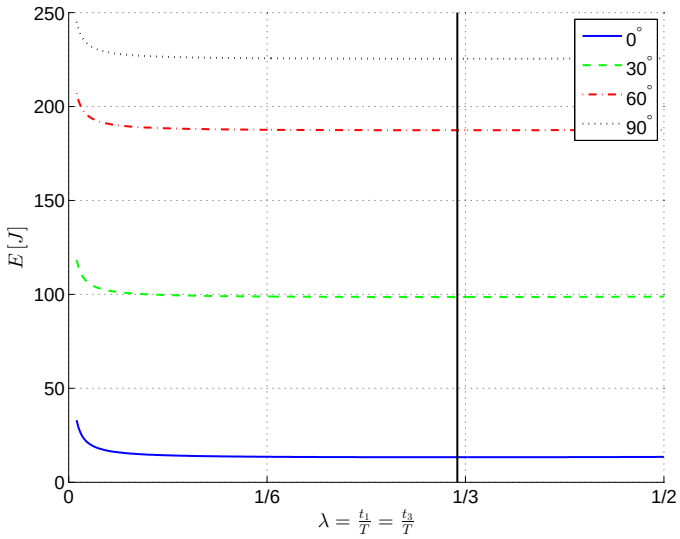


Figure 7: Trend of energy consumption at λ and ϑ varying, with $t_1 = t_3$ and $T = 3s$. The vertical black line represents the energy minimum value.

5 CONCLUSIONS

In this paper, starting from the results in [9], the issue of finding a minimum energy consumption for a point-to-point motion on a typical mechatronic system, i.e. a robotic linear axis, has been addressed. By taking into consideration the S-curve primitive, the expression of the total energy consumption for point-to-point motion has been formulated and computed. Then, the parameters that achieve minimum energy consumption, given the possibility to recover energy when braking, have been found in different cases either in a closed form or through a numerical solution. Future work will cover both the experimental validation of the method and the extension of the proposed solution to multi-axis systems.

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