Vibration suppression of speed-controlled robots with nonlinear control

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Abstract In this paper a simple nonlinear control strategy for the simultaneous position tracking and vibration damping of robots is presented. The control is developed for devices actuated by speed-controlled servo drives. The conditions for the asymptotic stability of the closed-loop system are derived by ensuring its passivity. The capability of achieving both improved trajectory tracking and vibration suppression are show trough experimental tests conducted on a three axes Cartesian robot. The control is aimed to be compatible with most industrial applications, given the simplicity of implementation, the reduced computational requirements and the use of joint position as the only measured signal.

Keywords Industrial robot, Nonlinear control, Vibration damping, Model-free control, Motion control

1 Introduction

Vibration reduction of high speed robot is a widely investigated topic, as testified by a vast literature [1] developed since the 70's. Despite the large effort made by the academia to study the influence of structural flexibility on robot dynamics [2,3], especially for the case of lightweight or flexible robots [4,5], the design of accurate and high-performance model-based control system as the main tool [6] for high-speed vibration-free operation of robots have failed to gain acceptance in industry, as remarked by several works [7,8] which state that 95% of control systems in use are of PID type and many of them are not even properly tuned [9]. For example, model-predictive control [10], intended as one of its many nuances such as GPC [11] or receding-horizon control [12] have proved to be very effective for achieving at the same time high speed and pronounced vibration damping [13,14]. In general the importance and the possible performance improvement brought by nonlinear and model-based strategies is well recognized by the roboticist community [15].

On the other hand, the definition, the use and the tuning of model-based control systems is not within the availability of many robotic practitioners. For this reason, alternative approaches such as the development of model-free controls and suitable trajectory planning algorithms [16] should be of less troublesome acceptance for the robot practitioner. Other approaches that have proved to be effective include reference shaping [17,18], i.e. the use of properly tuned convolution filters to process the references signal of input commands with the aim of limiting jerk [19] or avoiding to excite the resonant modes of the plant, also by taking into account the possibility of handling parametric mismatches in a robust framework [20,21].

Other researchers have proposed the use of non-time based strategies, among which the most prominent solution is the Delayed Reference Control (DRC), proposed by Gallina and Trevisani [22]. This techniques, which is based on the use of a variable time delay commanded by one or more feeback signal to the signals to be tracked, have proved to be very effective while being of simple application. The same technique has found use for multi-d.o.f. systems as well in [23,24] and haptic devices [25,26].

The aim of this paper is to introduce a simple nonlinear control strategy that is shown to be very effective for the simultaneous position control and vibration damping of industrial robots, which is of straightforward implementation, that has limited computational requirements and therefore could be potentially made available to standard industrial control systems without having to perform significant modifications to them. This control strategy has two fundamental characteristics: the control is model-free and it requires just one measured signal.

The first characteristic is fundamental in all industrial applications, in which the required knowledge to operate or to develop a dynamic model of the machine in use in not available. Also the limited number of measured values can actually widen the field of application of the proposed controller. In particular, the formulation presented here uses as the feedback signal just the position read from a quadrature encoder. Such signal, despite being a quantified signal, can be read by a simple digital circuit commonly available in most PLCs, and being of digital nature, is often unaffected by noise, unlike speed measurements. Speed measurement, often performed by tachometers, can be affected by significant amount of noise, leading to measurement errors in the high frequency range that can even jeopardize the closed-loop stability [27,28].

The paper is outlined as follows: the first section introduces the development of the proposed nonlinear control strategy, together with a definition of its stability bounds obtained through Lyapunov techniques. Section 2 shows the application of the proposed solution in a practical case: the controlled is used to experimentally evaluate the capability of the controller to achieve fast reference tracking and effective vibration damping when following a step reference on a single axis of a Cartesian robot. The third section reports the outcome of further tests involving the simultaneous motion of all three axes while following a smoother trajectory.

2 Nonlinear control design

The control design proposed and tested in this work can be used for the independent joint position control of a robot. In particular the control action is designed with a particular reference to robot whose actuator are speed-controlled, as for the device used for the experimental tests whose results are presented here. The manipulator used for the experimental tests on the proposed nonlinear control uses Siemens Simodrive 611 servo-drives set up for speed control: by using this configuration the servo-drive provides to the motor the electrical power needed to follow a speed profile which is described by an externally-supplied analog voltage reference, according to figure 1.

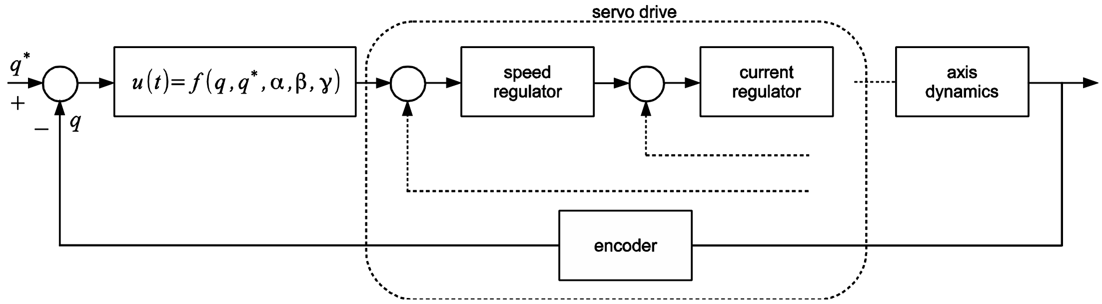

Fig. 1 Block diagram schematics of each axis of the robot

The proposed control is based on the following control action:

$$u(t) = \alpha\left(q^* - q(t)\right) + \beta\left(q^* - q(t)\right)^3 + \gamma \int_0^t \alpha\left(q^* - q(t)\right)dt \tag{1}$$

being q^* the reference for the angular position of the mechanism q^*. α, β and γ are fixed scalar quantities that represent the tuning parameters of the controller. The control action is nonlinear, given the presence of the term $\beta\left(q^* - q(t)\right)^3$. This control is especially designed to work with speed-controlled mechatronic systems, and therefore the control action is proportional to the speed instantaneously produced by the actuator. By setting the bandwidth of the speed control to be sensibly higher than the bandwidth of the custom designed position control, the influence of the inner control loop can be neglected during an analysis of the closed-loop dynamic behavior of each joint of the robot [29].

The global stability of the control action $u(t)$ expressed in eq. (1) can be assessed through Lyapunov techniques and its asymptotic stability can be ensured trough the use of Barbalat's Lemma [30]. A Lyapunov function candidate is chosen as:

$$V(t) = T(t) + U(t) + \frac{\gamma}{2}J\left(q^* - q(t)\right)^2 \tag{2}$$

If the control weight γ is positive, $V(t)$ is a non-negative function, being the sum of thee non-negative terms including the kinetic energy $T(t)$ and the potential energy $U(t)$ of the robot. J is the equivalent inertia of the mechanism as measurable at the actuator's revolution axis. Therefore $V(t)$ is a lower-bounded function. The time derivative of $V(t)$ is:

$$\dot{V}(t) = \dot{T}(t) + \dot{U}(t) - \gamma J\dot{q}(t)\left(q^* - q(t)\right) \tag{3}$$

If friction and internal dissipations are neglected, the variation of the total energy of the system over a time interval ΔT is due solely to the work done by the actuator over the same time interval:

$$T(t+\Delta t) - T(t) + U(t+\Delta t) - U(t) = \int_t^{t+\Delta t} \tau(s)\dot{q}(s)ds \tag{4}$$

in which $\tau(t)$ is the torque produced by the actuator. Taking the time derivative of eq. (4) leads to:

$$\dot{T}(t) + \dot{U}(t) = \tau(t)\dot{q}(t) \tag{5}$$

Now the time derivative of the Lyapunov candidate function can be rewritten as:

$$\dot{V}(t) = \tau(t)\dot{q}(t) + \dot{U}(t) - \gamma J\dot{q}(t)\big(q^* - q(t)\big) \tag{6}$$

By representing the dynamics of the machine as a single inertia driven by a torque-controlled actuator as:

$$J\ddot{q}(t) = \tau(t) \tag{7}$$

equation (6) can be rewritten as:

$$\dot{V}(t) = J\ddot{q}(t)\dot{q}(t) - \gamma J\dot{q}(t)\big(q^* - q(t)\big) \tag{8}$$

Now, being the actuator controlled in speed, under the condition that the control can achieve good speed tracking, the acceleration $\ddot{q}(t)$ is equal to $\dot{u}(t)$, with:

$$\dot{u}(t) = -\alpha\dot{q}(t) - 3\beta\dot{q}(t)\big(q^* - q(t)\big)^2 + \gamma\big(q^* - q(t)\big) = \ddot{q}(t) \tag{9}$$

By direct substitution of eq. (9) into eq. (8), $\dot{V}(t)$ can be rewritten as:

$$\dot{V}(t) = -\alpha J\dot{q}(t)^2 - 3\beta J\dot{q}(t)^2\big(q^* - q(t)\big)^2 \tag{10}$$

Therefore the first condition imposed by Barbalat's lemma is met when $\dot{V}(t) \le 0$ or, equivalently:

$$\alpha + 3\beta\big(q^* - q(t)\big)^2 \ge 0 \tag{11}$$

When met, the condition of eq. (11) ensures the simple stability of the system defined by closing the loop using the proposed nonlinear controller of eq. (1).

A direct analysis of eq. (1) shows that for $\beta = 0$ the proposed control act as a pure proportional-integral control, with gains γ and α. When $\beta \neq 0$, the term $\beta\big(q^* - q(t)\big)^2$ act as an additional proportional action to the one brought by α, which can be weighted by the amplitude of the tracking error.

The effects of a positive value of β is to increase the proportional control action set by the weight α, and therefore the bandwidth of the controller, when the tracking error is large. On the other hand, a negative value of β allows to weaken the aggressiveness when the tracking error is large. This feature can be adopted to reduce the vibration damping when a flexible system is subjected to high accelerations, as will be shown experimentally by the results presented in this work. Caution must be paid in order not to choose a too large negative value for β, especially when an higher bound on the tracking error $q^* - q(t)$ cannot be estimated with confidence.

In these situations an additional saturation effect can be added to the control action to force a lower bound on the term $\alpha + 3\beta\big(q^* - q(t)\big)^2$, thus ensuring the stability of the closed-loop system. Anyway, the test reported in this paper make use of a negative value for β, as this choice has lead to performance improvement, but it is not to be a-priori excluded that under some circumstances the use of a positive value for β might be beneficial to the closed-loop behavior of the system.

2.1 Asymptotic stability

Barbalat's lemma also requires for $\dot{V}(t)$ to be uniformly continuous, in order to ensure the asymptotic stability of the closed-loop system. A sufficient condition for the uniform continuity of $\dot{V}(t)$ is the boundedness of $\ddot{V}(t)$, according to [31], which can be proved here. According to eq. (10), under a proper choice of the tuning parameters, the system is dissipative, i.e.:

$$V\big(x(t)\big) \leq V\big(x(0)\big), \qquad \forall t \geq 0 \tag{12}$$

Also $V(t)$ is the sum of non-negative terms if $\gamma \geq 0$: therefore the boundedness of $V(t)$ implies the boundedness of all the three terms on the right-hand side of eq. (2).

The boundedness of the kinetic energy $T(t)$ also implies the same property to joint speed $\dot{q}(t)$ and, as a direct consequence, of $u(t)$ and of $\ddot{q}(t)$, according to eq. (9). Now $\ddot{V}(t)$ is:

$$\ddot{V}(t) = -2\dot{q}(t)\ddot{q}(t)\left(\alpha + 3\beta\big(q^* - q(t)\big)^2\right) + 6\beta\big(q^* - q(t)\big)\dot{q}(t)^2 \tag{13}$$

According to the last equation, the boundedness of $\ddot{q}(t)$ implies the boundedness of $\ddot{V}(t)$ and, as a consequence, the uniform continuity of $\dot{V}(t)$. By showing that the conditions imposed by Barbalat's lemma are met, the control action $u(t)$ defined in eq. (1) is asymptotically stable if suitable gains α, β and γ are chosen.

It should also be highlighted that, by neglecting the dissipative action of friction in eq. (4), the conditions for stability are actually evaluated in a conservative manner. For this reason the dissipative action of the controller will, in real situations, act together with the inherent dissipative action brought by frictions, with the last one working as an additional dissipative source neglected in the analysis above. Therefore the stability limits expressed in eq. (11) can possibly be violated without leading to instability in real situations.

3 Experimental results: step tracking

The closed-loop controller has been tested using the 3 axes Cartesian robot shown in figure 2. As already mentioned, the Cartesian robot sports three independent drive, i.e. one for each joint. The motion of each axis is deputed to a Siemens brushless motor, driven by Siemens Simatic 611 driver. The kinematic chain include three linear axes driven by a backlash-free reduction gear and a timing belt transmission systems.

The motor drives are set-up in order to receive an externally generated analog voltage speed reference signal. Such signal is provided by a control developed using LabVIEW and executed in real-time on a PXI-8110 controller. The controller can measure the angular position of each motor shaft using a quadrature encoder. The outer control loop, according to the block diagram of figure 1, runs at 1 kHz refresh frequency. The level of vibrations induced to the robot during motion is recorded using a PCB M352C65 piezometric accelerometer. Such device can measure the acceleration along a single axis up to ± 5 g. The accelerometer is mounted on the end-effector of the robot, measuring the acceleration along the Y axis, according to the notation of fig. 2. The Z axis is placed close to the minimum height for all the tests, in order to enhance the flexibility of the Z axis arm. In this configuration the Z axis arm acts as a cantilevered beam that can flex during motion, especially when high acceleration and jerks are involved [32].

The first test investigates the capability of tracking a position step reference signal by the proposed controlled, as well as its influence on the elastic behavior of the robot. This test is performed using only the Y axis of the robot. The controller is compared to a PI controller, in order to establish a comparison with the most widespread solution used in industrial applications [33].

Table 1: Step reference tracking: tuning parameters

PI control	Nonlinear control

$k_p = 0.08$	$\alpha = 0.22$
$k_i = 0.02$	$\beta = 2 \cdot 10^{-5}$
	$\gamma = 0.01$

The tuning parameters used for the first tests are reported in table 1. The position tracking performances of the two controllers are compared in figure 3. The proposed nonlinear control clearly achieves a faster and more accurate tracking of the step signal, while inducing a significantly lower amount of induced vibrations on the end-effector, as shown in figure 4.

The superior performance of the nonlinear control can be explained by considering the drawbacks normally encountered by PI control: this control requires a fairly high value of proportional gain k_p in order to achieve high bandwidth and fast tracking, but the higher such gain is, the higher are the induced vibrations during the transient. Also integral gain must be kept low, in order to avoid a pronounced overshoot. This drawbacks are by some degree overcome by the proposed nonlinear control: the availability of the nonlinear action $\beta\left(q^* - q(t)\right)^2$ allows to modulate the proportional action. In this sense, the choice of a negative value for β allows to reduce the proportional action during the first phase of the transient, during which the tracking error is large. As the tracking error converges to zero, the nominal value of the proportional action is restored for maximum disturbance rejection, with the additional benefit of allowing to "discharge" the integral action. This behavior can be noticed by analyzing the profiles of the control action generated by the two controllers in figure 4. From the figure it can be clearly seen that the nonlinear action prevents a sharp peak of control action at the very first beginning of each transient. The amplitude of this peak is directly proportional to the proportional gain k_p, whose value cannot be set too low to prevent a slow tracking.

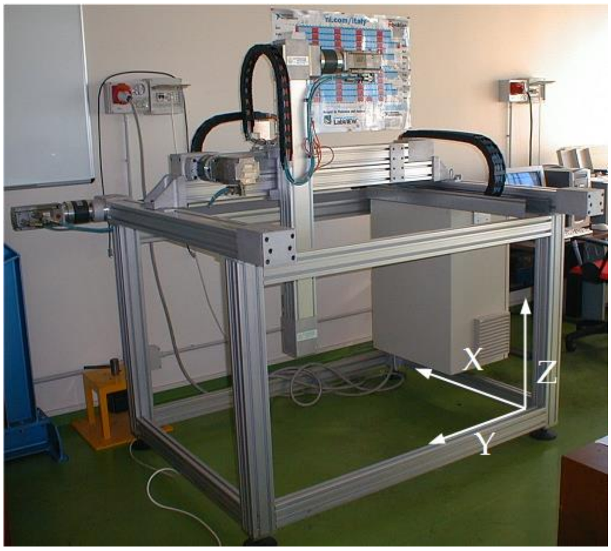

Fig. 2 The 3 axes Cartesian robot used for the experimental tests

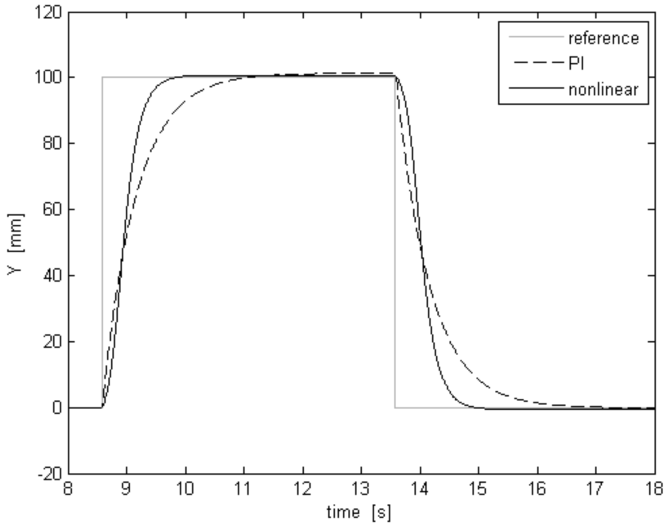

Fig. 3 Step tracking: comparison between PI and nonlinear control

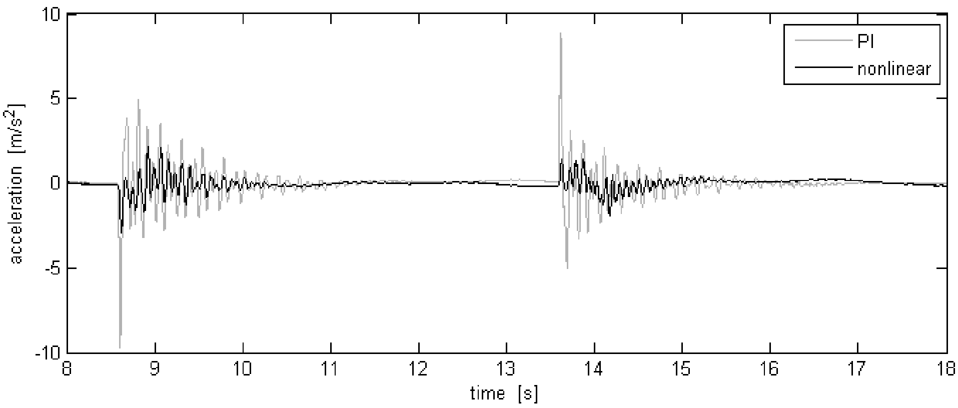

Fig 4 Step response: measured acceleration on the end-effector, comparison between PI and nonlinear control

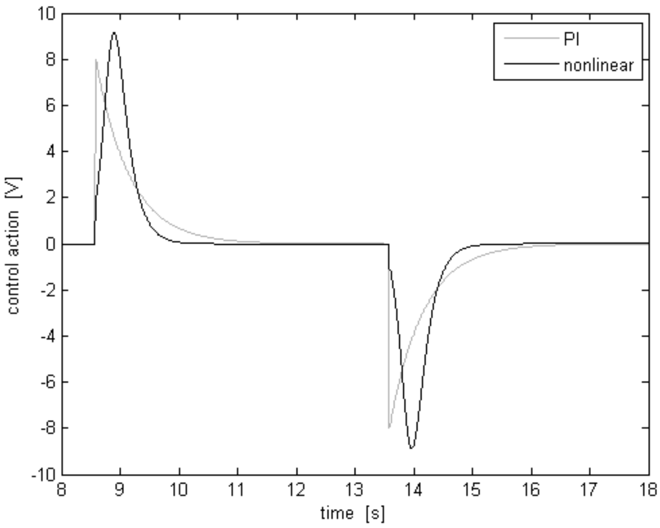

Fig 5 Step response: control action u, comparison between PI and nonlinear control

5 Experimental results: trajectory tracking

The experimental results presented in the previous section have highlighted the superior behavior of the proposed nonlinear control action in comparison with the common PI control when tracking fast changing references. In this section the nonlinear control is tested during the tracking of a smoother spatial trajectory, in order to prove that the proposed solution can be beneficial, in terms of vibration damping, also when smoother trajectories are involved. It is well known that a smoother trajectory, especially the ones with limited and continuous jerk [34], produces a lower level of induced vibrations.

The results presented here show that the nonlinear control can reduce the induced vibration while retaining nominally the same trajectory tracking capability. A minimum-time trajectory that passes through 13 via points have been planned, using the "434" trajectory primitive [35]. Such trajectory is composed by a sequence of 4th and 3rd order polynomials in time, in order to achieve continuous accelerations and limited jerk values. This kind of trajectory allows to achieve a good trade-off between the overall smoothness and the total execution time.

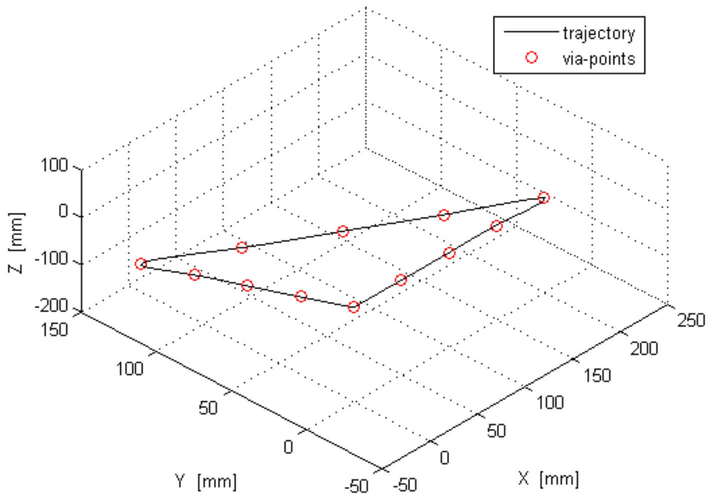

Fig 6 Trajectory in the operative space

The planned trajectory in the operative space is shown in figure 6. The via-points have been chosen in order to produce a triangular path that involves the motion along all three axes of the robot.

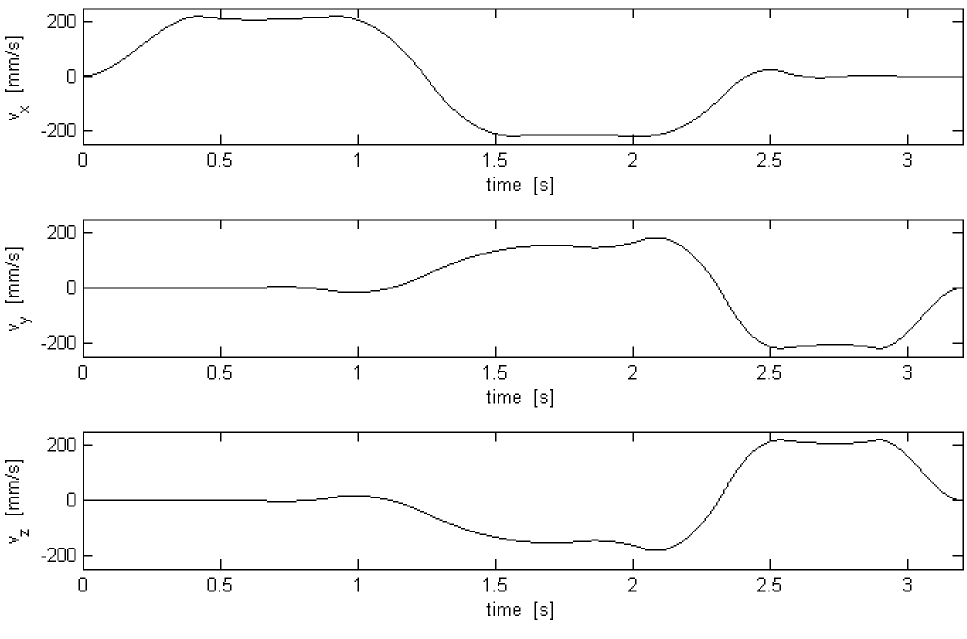

Fig 7 Trajectory: speed of X,Y and Z axes

The planning algorithm of choice allows to compute the minimum-time solution trajectory under kinematic constraints: in the case under investigation here the trajectory is generated in order to keep each axis speed below 220 mm/s, which is the maximum speed achievable by the robot. The speed profiles for the three axes are shown in figure 7, while the corresponding accelerations are shown in figure 8.

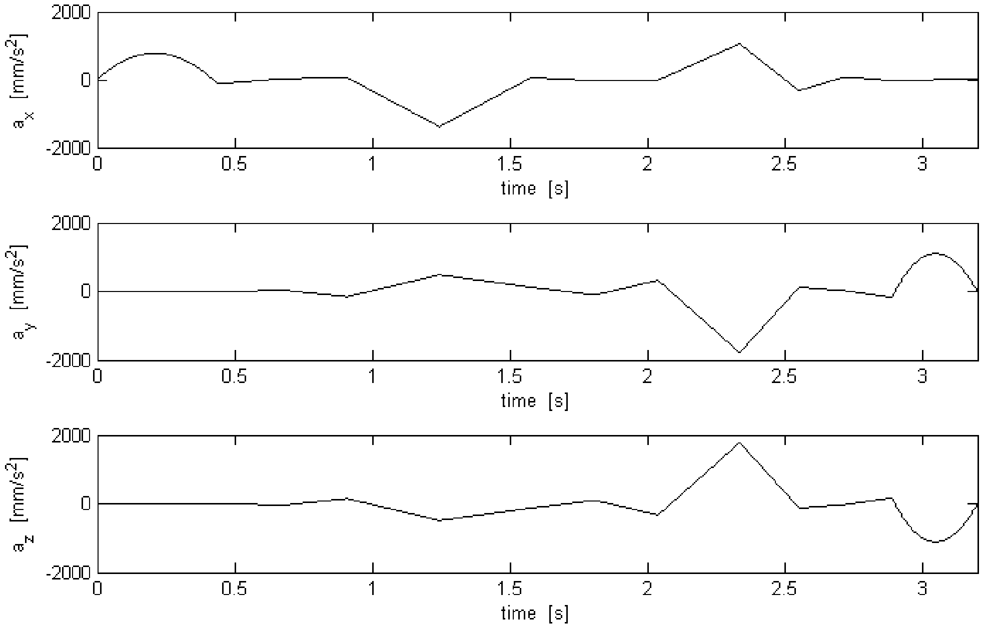

Fig 8 Trajectory: acceleration of X,Y and Z axes

The test, which compare the performance of the proposed controller with the one of a PI control, has been performed using the tuning parameters reported in table 2. The trajectory tracking performance for the three axes is very similar for the two controllers: according to figures 9-11 and the dashed black line which represents the results obtained with the nonlinear controller overlap almost perfectly the gray line, i.e. the trajectory followed by each axis with the use of PI control. In other words both controllers can achieve virtually the same trajectory tracking performance.

Table 2: Trajectory tracking: tuning parameters

axis	PI control	nonlinear control
X	$k_p = 0.2;\ k_i = 0.005$	$\alpha = 0.2;\ \beta = 0;\ \gamma = 0.005$
Y	$k_p = 1.5;\ k_i = 0.01$	$\alpha = 1.5;\ \beta = -0.005;\ \gamma = 0.01$
Z	$k_p = 1.5;\ k_i = 0.01$	$\alpha = 1.5;\ \beta = -0.004;\ \gamma = 0.001$

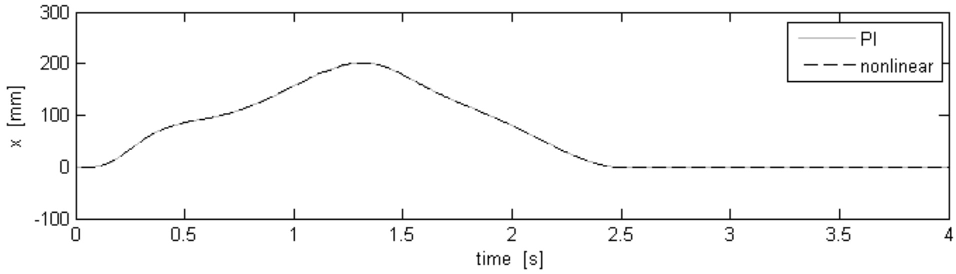

Fig 9 Trajectory tracking: end-effector position along the X axis: comparison between PI and nonlinear control

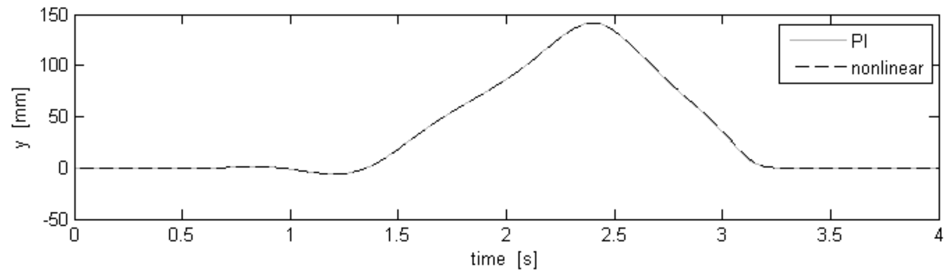

Fig 10 Trajectory tracking: end-effector position along the Y axis: comparison between PI and nonlinear control

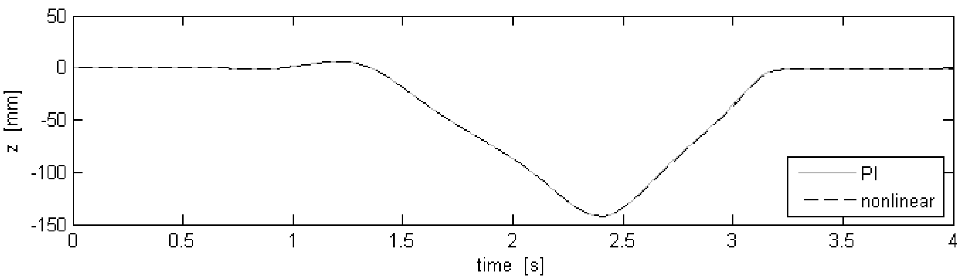

Fig 11 Trajectory tracking: end-effector position along the Z axis: comparison between PI and nonlinear control

As far as vibration damping is concerned, the use of the nonlinear controller allows to reduce by a notable amount the amplitude of the elastic vibrations measured on the end-effector of the robot. As for the previous experiment, an accelerometer is mounted on it to monitor the acceleration along the Y axis.

The recorded data are shown in figure 12. A detailed analysis of the results show that the measured level of vibration is very similar up to t = 7.8 s: before this time the motion happens exclusively along X axis, which, according to table 2 is controlled with $\beta = 0$, i.e. with a standard PI control. As soon as the speed of the two other axes is large, i.e. between t=7.8 s to t=9.8 s, the action of the nonlinear controller effectively reduces the amplitude of the vibrations measured along the Y axis. The results of this experiment therefore allows to conclude that, with the proper choice of tuning parameters, the proposed nonlinear controller can reduce the level of induced vibrations on the structure of the controller during high-speed 3D motion, while achieving good trajectory tracking performances.

Trough a comparison between the results presented in this section and the ones from the previous one, it can be inferred that the nonlinear controller, by being able to modulate the proportional action, leads to a vibration damping improvement that is inversely proportional to the smoothness of the trajectory. On the other hand, smoother trajectory are less prone to induce vibrations [16], and therefore when they are used the need for an pronounced vibration damping is less evident.

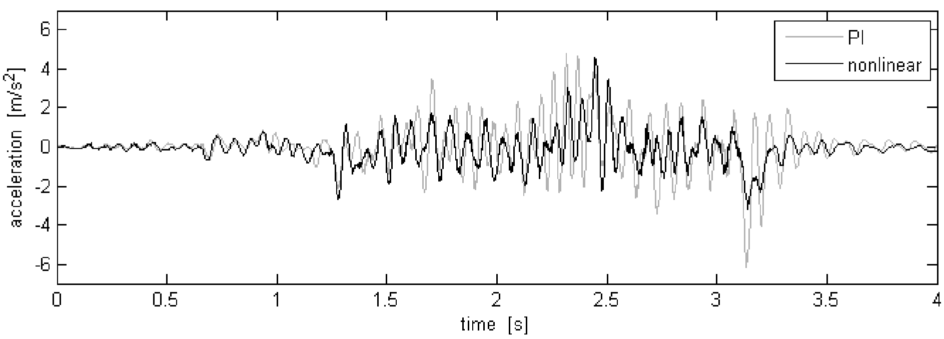

Figure 12: End-effector acceleration along the Y axis: comparison between PI and nonlinear control

Conclusion

In this paper a novel solution for the simultaneous trajectory tracking and vibration control for industrial robot is introduced. The proposed solution is based on a nonlinear control action, whose stability bounds are evaluated using Barbalat's lemma and Lyapunov techniques, introducing the conditions for simple and asymptotic stability. The control is applied to a 3 axes Cartesian robot operated by speed-controlled actuators. Experimental results show that the proposed control, while being of straightforward application, of limited computational resource requirements, and despite the fact that elastic displacements are not measured, can reduce the amplitude of high-speed motion vibration while improving reference tracking for fast-changing references or while obtaining a very similar tracking capability for smooth trajectories in comparison with the traditional and ubiquitous PI control.

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