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Trajectory Following and Vibration Control for Flexible-link Manipulators

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Abstract This paper is aimed at showing the capabilities of the Model-based Predictive Control for simultaneous vibration suppression and trajectory following for a class of flexible-link multi-actuated manipulators. For this class the requirements for the control system are very strict, since any error on the end-effector trajectory can be amplified by the time-varying kinematic configuration of the manipulator. Moreover, an optimal synchronization between the movement of the two axes must be settled. The tests provided here have been developed through exhaustive numerical simulations, the results of which show the capabilities of the MPC controller even for multi-actuated compliant mechanisms.

1 Introduction

The modeling and control of flexible-link mechanisms are key issues in robotics engineering. Critical vibration due to inertial components of the motion arise in manipulators when they are exposed to large acceleration. These dynamic effects can lead to considerable worsening of accuracy, to mechanical failures, as well as instability. An accurate description of the dynamics of multi-link flexible mechanism requires complex and nonlinear models. The problem is even more challenging when trajectory tracking in the operative space is the main concern, since the accuracy requirements are much stricter. The purpose of this paper is to show the capabilities of the Model-based Predictive Control (MPC) strategy for trajectory control and vibration reduction for with regard of a class of multi-actuated flexible-link mechanisms. The use of MPC as a vibration controller has been investigated in a limited number of scientific papers. For example, in [1] the MPC has been applied as a vibration controller in a constrained beam, by means of piezo-electric actuators. In [2] the MPC has been used to control torsional vibration in a milling machine, while in [3] it has been

used to control a flexible-joint mechanism. Moreover, as far as the use of MPC for flexible-links mechanism is concerned, the literature is even less significant: to the authors' knowledge the only papers focusing on this field are [4–7]. In [4, 6, 7] predictive control strategies have been used to control position and the vibration of a single-link mechanism, while in [5] a constrained MPC has been applied as the position-regulator for a four-link closed-chain compliant mechanism. The present work is, therefore, an evolution of previously published studies [4, 5]. In particular, a 5-R planar mechanism will be employed to verify the performance of the MPC strategy for a class of multi-actuated flexible-link mechanisms. The 5-R mechanism has two rigid degrees of freedom and requires two actuators. The control problems increase with respect to the previous works, since the movement of the controlled axes must be more precise and synchronized. This causes the major constraints on the accuracy in each axis. The MPC controller has been implemented in software simulation using Matlab/ Simulink[®]. Exhaustive simulations have been conducted to prove the effectiveness of this control approach.

This paper is arranged as follows: Section 2 will briefly illustrate the MPC control strategy. The dynamic model of the 5-R flexible-link mechanism will be constructed in Section 3, while Section 4 will describe the overall control system. Finally, Section 5 will show and discuss the results of a suitable simulation.

2 Model-based Predictive Control

MPC control is based on the following basic ideas: receding-horizon strategy, the internal prediction model, and constraints on control action and manipulated variables. Owing to the length constraints of this paper, in this section only a brief explanation of the concepts mentioned above will be given. Therefore, for more details the reader should refer to [4, 5, 9–11]. The term MPC control refers to a class of optimal controllers. In the MPC controller, the cost-function depends on the instantaneous feedback signals, on the prediction of the future behavior of the plant and, finally, on the ideal trajectory (Figure 1.a).

The evaluation of the future behavior of the plant is obtained from an internal model of the plant, which is computed over a time interval called *prediction horizon* (H_p). The term *control horizon* (H_c) refers, on the other hand, to the length of the control sequence. By defining $\mathbf{z}(k)$ as the vector of the manipulated variables and $\mathbf{r}(k)$ as the vector of the instantaneous reference, the optimal control sequence $\mathbf{w}(k)$ is computed at every step by minimizing this cost-function:

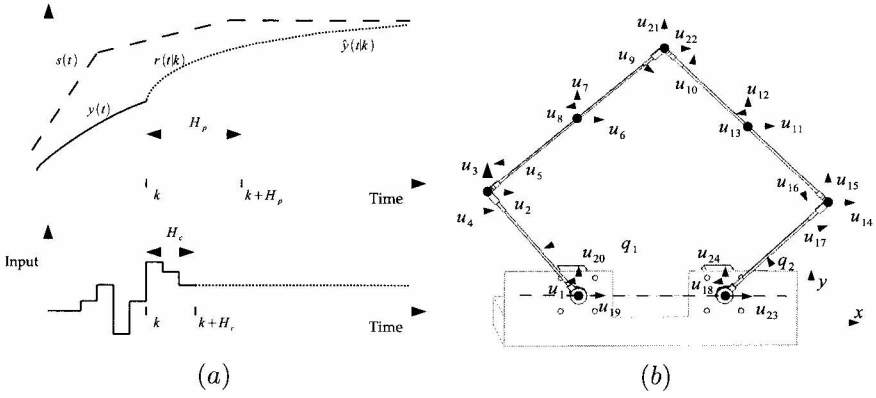


Figure 1. (a) Receding horizon strategy; (b) 5-R mechanism: elastic and rigid displacements

$$\mathcal{V}(k) = \sum_{i=1}^{H_p} \|\mathbf{z}(k+i|k) - \mathbf{r}(k+i)\|_{\mathbf{Q}}^2 + \sum_{i=0}^{H_c-1} \|\Delta \mathbf{w}(k+i|k)\|_{\mathbf{R}}^2 \quad (1)$$

It must be pointed out that the optimal control sequence is computed at every time step k . However, only the first element of this sequence is eventually fed back to the plant. \mathbf{Q} and \mathbf{R} are weighting matrices for the quadratic norm of the tracking error and control effort. A more detailed description of the use of MPC for flexible-link mechanisms can be found in [5].

3 Dynamic model

In this section the dynamic model of the flexible-link mechanism proposed by Giovagnoni [8] will be briefly explained. The choice of this formulation among the several proposed in the last 40 years has been motivated mainly by the high level of accuracy allowed by this model. Each flexible link belonging to the mechanism is subdivided into finite elements. The mechanism's motion can be thought as the superposition of the motion of an equivalent rigid-link system (ERLS) and the elastic motion of the nodes of the finite elements. Therefore, the independent coordinates of the system correspond to the angular position of the two cranks and the vector of the nodal displacement \mathbf{u} . The dynamic equation of motion is:

$$\begin{aligned}
& \begin{bmatrix} \mathbf{M} & \mathbf{M}\mathbf{S} & \mathbf{0} & \mathbf{0} \\ \mathbf{S}^T\mathbf{M} & \mathbf{S}^T\mathbf{M}\mathbf{S} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{q}} \\ \dot{\mathbf{u}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{I} \\ \mathbf{S}^T\mathbf{M} & \mathbf{S}^T \end{bmatrix} \begin{bmatrix} \mathbf{g} \\ \boldsymbol{\tau} \end{bmatrix} \\
& + \begin{bmatrix} -2\mathbf{M}_G - \alpha\mathbf{M} - \beta\mathbf{K} & -\mathbf{M}\dot{\mathbf{S}} & -\mathbf{K} & \mathbf{0} \\ \mathbf{S}^T(-2\mathbf{M}_G - \alpha\mathbf{M}) & -\mathbf{S}^T\mathbf{M}\dot{\mathbf{S}} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{q}} \\ \mathbf{u} \\ \mathbf{q} \end{bmatrix} \quad (2)
\end{aligned}$$

A detailed description of this model can be found in [8].

The 5-R mechanism is the 2DoFs manipulator shown in Figure 1.b. It is comprised of four steel rods connected in a closed-loop chain by using five revolute joints. The motion of the cranks is governed by two torque-controlled actuators. The fifth link (i.e., the chassis) can be considered to be perfectly rigid without affecting the accuracy of the model. The mechanical characteristics of the mechanism are shown in Table 1. The links are very thin and the whole mechanism is quite prone to vibration.

The overall FEM model is characterized by 24 nodal displacements, as it can be seen in Figure 1.b.

Table 1. The kinematic and dynamic characteristics of the flexible-link mechanism

	symbol	value
Young's modulus	E	200×10^9 [Pa]
Flexural inertia moment	J	1.08×10^{-10} [m^4]
Beam width	a	6×10^{-3} [m]
Beam thickness	b	6×10^{-3} [m]
Mass/unit of length of links	m	0.282 [kg/m]
Length of links 1-4	L_i	0.3 [m], 0.6 [m], 0.6 [m], 0.3 [m]
Ground length	L_5	0.3 [m]
Rayleigh damping constants	α	8.72×10^{-2} [s^{-1}]
	β	2.1×10^{-5} [s]

4 Control System

Figure 2 shows the control scheme. Since only a set of the state vector's components can be measured (i.e., the cranks' angular position and the corresponding elastic displacements), an Extended Kalman Filter is used to estimate the whole state from a subset of it. The estimated current state, as well as the previously generated inputs, are used to predict the future behavior of the mechanism and to optimize it. Eventually, the optimal sequence is computed and its first component input into the actuators.

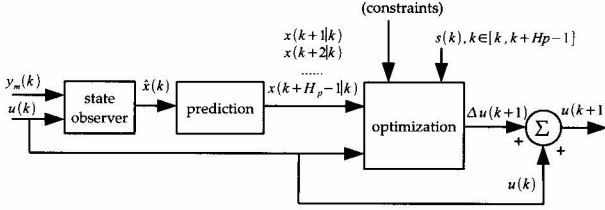


Figure 2. Control scheme

5 Simulation results: trajectory tracking

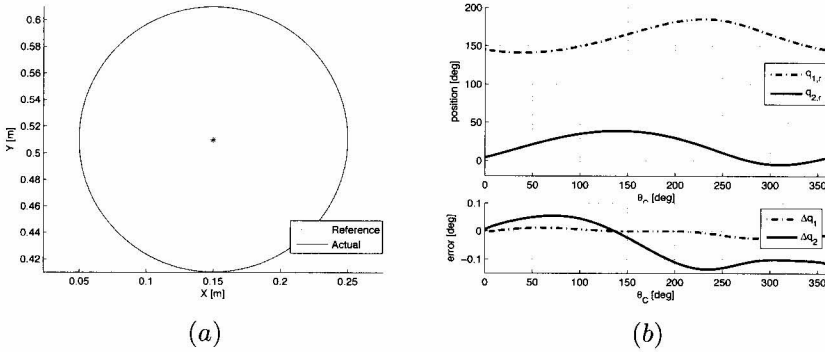


Figure 3. (a) Trajectory in the Cartesian space; (b) Trajectory and position errors of the joints

In this section, the effectiveness of the MPC as a suitable controller for multi-actuated flexible-link manipulators will be proven and discussed by investigating the behavior of the 5-R mechanism. The task consists of tracking the circumference with the center in $\mathbf{C}_c = [0.150m, 0.510m]^T$ and radius $r_c = 0.10m$: $C: \mathbf{X}(\theta_c) = \mathbf{C}_c + r_c [\cos \theta_c \quad \sin \theta_c]^T$, $\theta_c \in [0, 2\pi]$

The trajectory has been planned as in [12]. This procedure, indeed, allows for the computation of the optimal trajectory depending on jerk and execution time. Figure 3.(a) shows the trajectory in the Cartesian space, while Figure 3.(b) shows the trajectories of the related joints. The MPC controller has been tuned with the following values: $H_p = 5$, $H_c = 5$, $w_{u_1} = w_{u_{18}} = 2 \times 10^3$, $w_{q_1} = w_{q_2} = 4 \times 10^4$.

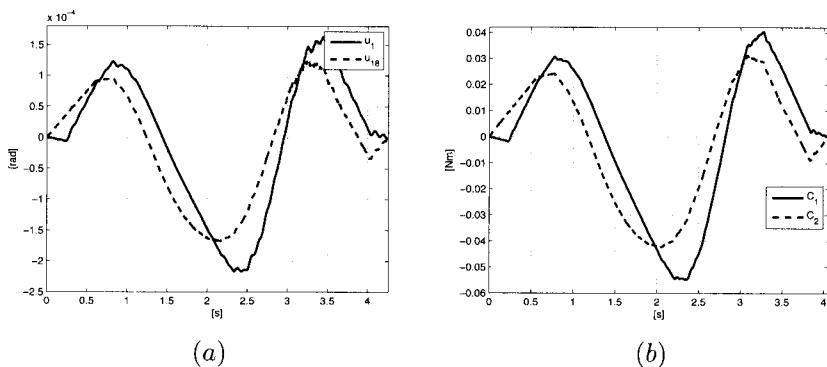


Figure 4. (a) Nodal displacements u_1, u_{18} ; (b) Input torques

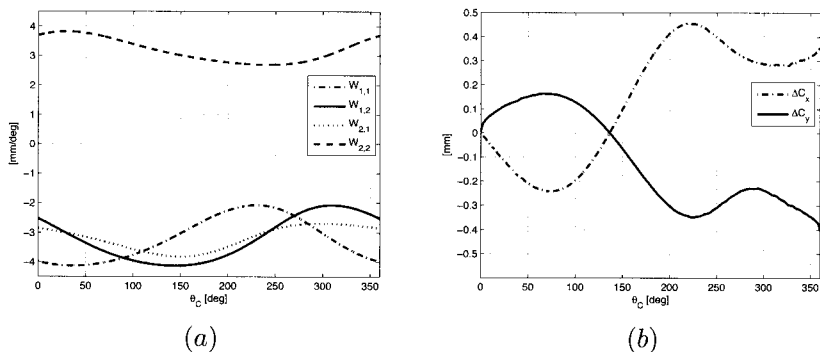


Figure 5. (a) Sensibility's coefficients; (b) Position error

The main simulation results are shown in Figures 3 through 5. In particular, Figure 3.b shows the angular positions of cranks q_1 and q_2 and their corresponding errors. It should be pointed out that the joint errors are never more than 1.2 tenths of degree, which proves the considerable performance of the controller. At the same time, the elastic displacements u_1 and u_{18} are shown in Figure 4.a. By comparing Figure 4.a with Figure 4.b (where the input torques are shown); it is possible to evaluate the capabilities of such a controller much more clearly. The vibration affecting the nodal displacements is, indeed, kept very minimal and the main displacements can be attributed largely to the accelerating torques input; therefore, the MPC controller is able to reduce vibration as well. As far as the position errors in

the Cartesian space are concerned, the MPC seems to be a little less high-performance (Figure 5.b). This can be imputed to the kinematic structure of the mechanism, as well as to the physiological behavior of the proposed controller. The kinematics of the manipulator, indeed, amplifies the joint errors as follows: $\Delta \mathbf{C} = \mathbf{S}_C(\mathbf{q})\Delta \mathbf{q} = [w_{ij}] \Delta \mathbf{q}$, where $\mathbf{S}_C(\mathbf{q})$ is the sensibility coefficients' matrix of the end-effector. It should be noted that this matrix depends on the current manipulator configuration. As such, the amplification of the joint errors varies as the manipulator changes configuration. Figure 5.a shows the values of the four elements of \mathbf{S}_C . By considering the joint errors of Figure 3.b and the coefficients of Figure 5.a, it is possible to infer the error of the end-effector shown in Figure 5.b. As a result, in order to reduce the position error in the Cartesian space, a high-performance control action must be settled on the joints. This issue is dependent on the physiological behavior of the controller, which can only handle the end-effector position through the kinematic structure of the manipulator. Neither does a position's feedback exist nor does the MPC controller have an internal kinematic model of the structure. The kinematic model would enormously complicate the controller and would require a much higher computational effort. A better solution for MPC control is currently being investigated by the authors. It will permit forcing the axes to behave in a more synchronized way. Although the resulting trajectory will be slightly slowed down, the position performance appears to be very promising.

6 Conclusion

In this paper a predictive control strategy has been proposed as an effective solution to the problem of simultaneous trajectory tracking and vibration suppression for compliant mechanisms with multiple actuation. The control system is based on receding horizon strategy, reference lookahead and an accurate prediction model. The mechanism chosen to validate, through extensive sets of numerical simulations, the effectiveness of the controller is a flexible five-link planar mechanism. The control system proved to be very effective in both trajectory tracking and vibration suppression, even in tasks encompassing high speed and extensive movement.

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