Robust rest-to-rest motion planning for cranes through a variational solution

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Abstract

The operation of overhead cranes requires to deal with the problem of damping or eliminating the load oscillation that naturally occur during and after the motion. Such a problem is usually tackled both in literature and in industrial practice through closed-loop control or through open loop control. The latter approach consists in designing optimal command profiles that move the load to the desired position without residual load oscillations. One of the most critical issues in the development of effective command references is the robustness with respect to the unavoidable model uncertainty. Despite its relevance, this problem has been explored less frequently.

An effective approach to include robustness specification in rest-to-rest motion planning of oscillatory systems is the one introduced and tested numerically in [1], which is for the first time validated experimentally in this work. The proposed solution is based on the formulation of the trajectory planning problem as a Two-Point Boundary Value Problem (TBPVP) to be solved through some well-established and reliable methods. In order to account for robustness, the proposed method is based on a new definition of the standard TBPVP by including the sensitivity functions of the dynamic model of the plant and by adding suitable additional boundary conditions. Additionally, some new features are included in the problem formulated in this work, to deal with the characteristics of real systems with finite control bandwidths.

In its nominal form, a rest-to-rest motion planning problem can be translated into a variational problem as follows. The dynamics of the flexible system should be described by a system of first-order ordinary differential equations (ODEs), $\dot{\boldsymbol{x}}(t) = \boldsymbol{\Omega}(\boldsymbol{x}, u, t, \eta)$ in which $\boldsymbol{x}(t)$, $\boldsymbol{u}$, t and η are the state, the input, the time and a generic scalar model parameter, respectively. The properties of the trajectory can be shaped by choosing the suitable cost function, defined as the time integral of the function $\boldsymbol{f}(\boldsymbol{x}, t, \boldsymbol{u}, \eta)$ evaluated over the execution time, with desired initial and final states $\boldsymbol{x}(t_0)$ and $\boldsymbol{x}(t_f)$ as the boundary conditions. The solution of the TPBVP problem can be solved, in its nominal form, by defining the Hamiltionian $\boldsymbol{\mathcal{H}} = \boldsymbol{f} + \boldsymbol{\lambda}^T\boldsymbol{\Omega}$, where $\boldsymbol{\lambda}(t) = [\lambda_1, \dots, \lambda_N]^T$ are the Lagrangian multipliers. According to the Pontryagin Minimum Principle, the necessary conditions for the optimal solution are:

$$\frac{\partial \boldsymbol{\mathcal{H}}}{\partial \boldsymbol{u}} = \boldsymbol{0};\ \dot{\boldsymbol{x}}(t) = \frac{\partial \boldsymbol{\mathcal{H}}}{\partial \lambda};\dot{\boldsymbol{\lambda}}(t) = \frac{\partial \boldsymbol{\mathcal{H}}}{\partial \boldsymbol{x}} \tag{1}$$

The conditions in Eq. (1) can be written as a single equation by defining the minimizing Hamiltionian $\mathcal{H}^* = \boldsymbol{\mathcal{H}}(\boldsymbol{u}^*(t))$, with $\boldsymbol{u}^*(t)$ as the input such that $\frac{\partial \boldsymbol{\mathcal{H}}}{\partial \boldsymbol{u}} = \boldsymbol{0}$. The minimizing Hamiltionian can be used to define a system of ODEs with the augmented state vector $\boldsymbol{y}(t) = [\boldsymbol{x}, \boldsymbol{\lambda}]^T$:

$$\dot{\boldsymbol{y}} = \begin{bmatrix} \frac{\partial \mathcal{H}^*}{\partial \lambda} \\ -\frac{\partial \mathcal{H}^*}{\partial \boldsymbol{x}} \end{bmatrix} \tag{2}$$

The solution in Eq. (2) is the nominal solution for the rest-to-rest motion planning problem, since the effects of the perturbation to the parameter η are neglected.

The robust motion planning problem can be defined by augmenting the dynamic model with the sensitivity functions of $\boldsymbol{\Omega}$, i.e. its partial derivative with respect to η, leading to the augmented system $\boldsymbol{\Omega}_\mathrm{r}$ and state $\boldsymbol{x}_r$:

$$\dot{\boldsymbol{x}}_r(t) = \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{s}}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}, t, \eta) \\ \frac{\partial \boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}, t, \eta)}{\partial \eta} \end{bmatrix} = \boldsymbol{\Omega}_\mathrm{r}(\boldsymbol{x}, \boldsymbol{u}, t, \eta) \tag{2}$$

The robust problem can be solved through the method described by Eqs. (1) and (2), with obvious modifications of the meaning of the terms. Some additional boundary conditions can be set. In particular, the values of the sensitivity functions $\boldsymbol{s}(t)$ are set to zero at both initial and final time to impose minimal sensitivity of the solution to possible mismatches between the real and the modeled plant.

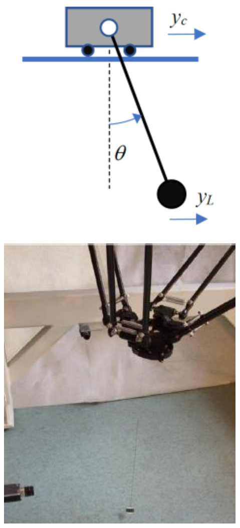

Figure 1. Scheme and picture of the studied system

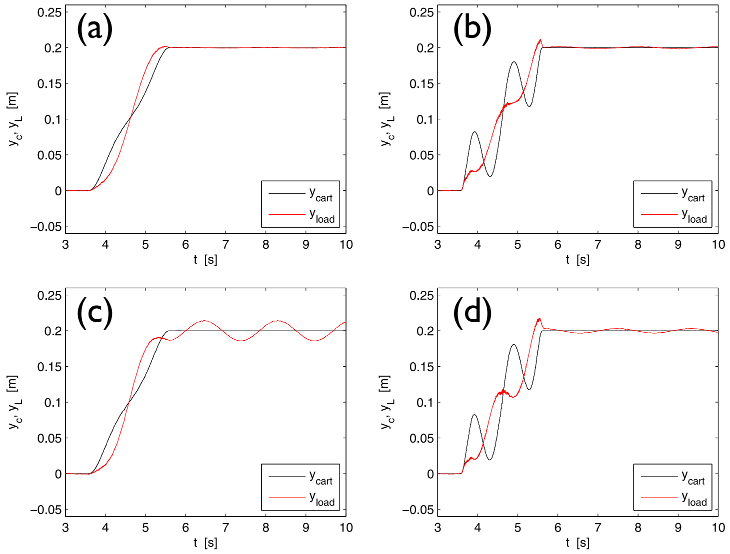

Figure 2. Experimental results: nominal (a,c) and robust trajectories (b,c), with nominal (a,b) and perturbed plants (c,d)

The model can be further augmented to include simplified models of the controlled actuator dynamic and hence to ensure feasibility of the generated motion profiles in real systems. Additionally, unlike other works in literature (see e.g. [2]), it can be applied to plants described by nonlinear dynamics, with both forces or accelerations as the model input.

Experimental results have been obtained using the testbed shown in Fig. 1, in which an Adept Quattro robot is used to mimic the motion of the cart of an overhead crane. A massless pendulum with length equal to 0.962 m is attached to the cart and the angular position of the pendulum θ is measured using a camera. The experimental results shown in Fig. 2 compares the nominal and robust trajectories synthesized through the proposed method. All measurements refer to a sample motion consisting in a translation of the cart equal to 0.2 m in 2 s. The output of the problem solution is the time history of the desired position for the cart, which is used as the command references for the robot closed-loop position control.

The plot in Fig.2(a) shows the results of the application of the nominal trajectory to the nominal plant with no model mismatch, by showing the measured position of the cart y_c and the measured position of the suspended load y_L. The plot (b) shows the results obtained by synthesizing the command reference with the robust approach and, again, with the nominal plant. In particular, the robustness improvement has been obtained by including the sensitivity function with respect to the natural frequency of the load swing vibrational mode. In both cases the prescribed rest-to-rest conditions are obtained, since the residual oscillation of the load is negligible. The slightly lesser accuracy obtained in the case of robust planning is mainly due to the limited tracking precision of the robot observed during high dynamic motion. It should be noticed that the nominal trajectory, as shown in Fig.2(a), has only positive values of the cart speed. In contrast the robust command reference in Fig.2(b) imposes both positive and negative speeds. This feature results in higher peak values of cart speed and acceleration, and in harmonic components with higher frequency, thus making the tracking more difficult.

In order to test the actual robustness improvement brought by the proposed method, two further results are shown in Fig.2(c-d). Such tests consist in performing the same motion with a pendulum length reduced by 0.18 m. This modification induces an increase of the natural frequency of the load oscillation and therefore a sensible mismatch between the modeled and the actual plants. As can be seen in Fig.2(c), the application of the nominal trajectory results in a large residual oscillation of the load, while a noticeably lower amplitude is obtained by using the robust trajectory. This result confirms that the proposed method can be effectively used to improve the robustness to parametric mismatches between the model of the plant used for trajectory planning and the actual plant.

Other results, not reported here, has shown that the proposed techniques can effectively plan much faster motion profiles, lasting less than half of the oscillation period, and can be extended to overhead cranes with double and triple pendulum as well.

References

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