Spring design for motor torque reduction in articulated mechanisms

Paolo Boscariol, Giovanni Boschetti, Paolo Gallina, Chiara Passarini

Abstract In this paper a procedure to reduce the motor size in articulated mechanism is developed. In particular, the paper presents a method that allows reducing the peak torque requirement of a single degree of freedom mechanism, through the use of an elastic element. Unlike other works in literature, in this work the choice of the trajectory primitive and the resulting inertial effects are taken into account. Both numerical and experimental results show that the inclusion of a single spring can sensibly decrease the peak torque requirements, thus allowing a cost-effective design modification.

1 Introduction

The design of mechanisms and of automatic machine is often aimed at achieving lightweight, energy efficient and cost-effective solutions. In particular, when dealing with automatic machines and robots, a large amount of the production costs is due to the actuation system. In this sense, a sensible cost reduction can be achieved by reducing the size of the motor, i.e. by reducing the torque requirements for a given task.

The same design objective is common to a widely investigated topic, which is the gravity balancing of mechanism. According to the common definition, a machine is said to be gravity balanced if the mechanism can be kept in equilibrium at any configuration without any actuator effort . It is clear that in general, a gravity balanced mechanism is effectively more energy efficient than a non-equilibrated one. This

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topic has been investigated in a large number of works. Some notable example can be found in the design of orthosis devices, in which springs and auxiliary kinematic chains are added with the aim of facilitating the motion of physically impaired patients [1]. A similar concept is investigated also in [10], which proposes solutions to improve the feasibility of the resulting design, by adding only springs. The problem of static balancing of mechanism is also investigated in [6], which examines the problem of gravity balancing from a wider perspective, by suggesting and analyzing several design modifications.

Another approach that uses springs to increase the efficiency of mechatronic systems is the application of natural motion, which consists on the use of elastic elements to reduce the power consumption of robots in repetitive operations, obtained by proper shaping of the potential elastic energy. With this approach, the acceleration of the mechanism is achieved by the release of the previously stored elastic energy, which is transformed into kinetic energy. Conversely, the deceleration happens with the opposite energy transformation. In this approach additional actuated joints are used to adapt the mechanism to different trajectories. Recently in [2], an optimization procedure, which involves the computation of the inverse multibody dynamics of a five-bar linkage, shows the effective energy reduction for pre-defined periodic motions. The added resonance induced by an additional spring is exploited in [8] as well, which shows a 56 % energy consumption reduction for a planar RR mechanism. In a similar way, by also actively controlling the equivalent stiffness, the power consumption of a single joint mechanism [7] and of a scara robot [5] can be considerably reduced.

This works follows a different path, i.e. it is aimed at evaluating the possible reduction of the torque requirements in planar articulated mechanisms trough the inclusions of springs with appropriate characteristics and appropriate connection points, and therefore without the addition of auxiliary actuated joints or links. The proposed solution takes into account the choice of the trajectory that is chosen for the operation of the mechanism, therefore showing also the effect of the trajectory choice on the resulting performance. It also well known that the choice of the trajectory has a strong impact on the actuator effort [3] and on the power consumption of mechatronic systems as well [9, 4]. To the best of authors' knowledge, there are no methods available in literature that directly relate the design of springs to reduce motor sizing to the particular trajectory chosen for the operation of the mechanism.

The paper is organized as follow: The first part describes the theoretical formulation of the torque minimization problem and some numerical results, while the second part shows the results of the experimental verification of the proposed solution.

2 Dynamic modeling

The mechanism considered in this document is a single link with one degree of freedom and constant inertia, actuated by a rotating motor, as shown in Fig. 1. The

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proposed method makes use of an optimization algorithm aimed at minimizing the peak torque requirements by choosing an optimal choice of a spring and of its connection points. The optimization is performed on the basis of the inverse dynamics of the mechanism, which allows to compute the required torque according to the spring design and to the chosen joint trajectory.

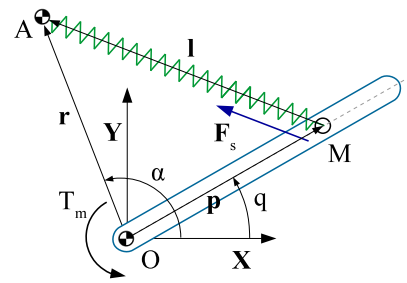

Fig. 1 Kinematic model of the mechanism

The choice of a motion law provides the time evolution of $q(t)$, $\dot{q}_t$ and $\ddot{q}(t)$, which are respectively the position, the velocity and the acceleration of the degree of freedom of the mechanism. The Newtonian formulation of the dynamics of the mechanism can be described as follow:

$$J\ddot{q}(t) = T_m(t) - \mu\dot{q}(t) + T_s(q,t) \tag{1}$$

in which T_m is the torque produced by the motor, the term $\mu\dot{q}$ accounts for viscous friction at the motor joint, while T_s is the torque contribution produced by the spring pull. Equation (1) is valid provided that the spring mass is negligible. If a Cartesian reference frame $\{\mathbf{X},\mathbf{Y},\mathbf{Z}\}$ is located about the actuated joint in O, the position of the point M can be represented by the vector $\mathbf{p} = p\,[cos(q), sin(q), 0]^T$, being the length of such vector, p, equal to the length OM. In the same way, the location of the point A is indicated by the vector $\mathbf{r} = r\,[cos(\alpha), sin(\alpha), 0]^T$. If the spring is represented by the vector $\mathbf{l}$, it can be expressed as $\mathbf{l} = \mathbf{r} - \mathbf{p}$, and its length, l as: $l = \|\mathbf{r} - \mathbf{p}\|$. The direction of the vector is represented as θ_s, while the unity vector associated with $\mathbf{l}$ is indicated as $\hat{\mathbf{l}}$. If the extension of the spring is oriented in the opposite direction form the one of $\mathbf{l}$, the force exerted by the spring, whose stiffness and free length are k and l_0, respectively, can be evaluated as:

$$\mathbf{F}_s(q) = k\,(l - l_0)\,\hat{\mathbf{l}}(q) = k\,(\|\mathbf{r} - \mathbf{p}(q)\|)\,\hat{\mathbf{l}}(q) \tag{2}$$

The torque produced by the spring pull $\mathbf{F}_s$ about the point O can be measured as:

$$T_s(q) = (\mathbf{p}(q) \times \mathbf{F}_s(q))\,\hat{\mathbf{z}} \tag{3}$$

or, in a more explicit form:

$$T_s(q) = k \cdot OM\,(\|\mathbf{r} - \mathbf{p}\| - l_0)\,sin\,(\theta_s - q) \tag{4}$$

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The angle θ_s can be evaluated as:

$$\theta_s(q) = atan2\left(rsin(\alpha) - psin(q), rcos(\alpha) - pcos(q)\right) \tag{5}$$

Equations (1) and (4) show that the equivalent elastic torque $T_s(q)$ can be used to modify the amplitude of motor torque T_m: ideally the motor torque could reach the zero value if a proper shaping of $T_s(q)$ allows to compensate for the inertial effects and for friction. Eq. (1) highlights also that the optimal design of the elastic torque T_s is affected by the time evolution of the joint variable, i.e. that for each choice of the joint trajectory profile a different optimal spring design exist.

3 Spring design optimization

The reduction of the motor size is performed through the definition and the solution of the following optimization problem:

$$\begin{aligned} \min_{q\in[0,2\pi]} \quad & \|T_m(r,\alpha,k)\|_\infty \\ & \text{subject to:} \\ & k > 0 \\ & r \geq (OM + l_0) \\ & 0 \leq \alpha < 2\pi \end{aligned} \tag{6}$$

The optimization problem of Eq. (6) defines the cost function to be the infinity norm of the motor torque, evaluated over a whole rotation of the mechanism. The decision variables include the distance r and the angle α, which are used to define the location of the fixed end of the spring, i.e. the point A. The value of the spring stiffness constant k is included among the decision variables as well.

In order to produce results that are compliant with the available experimental rig, the constraint $r \geq (OM + l_0)$ is included. This constraint ensure that the spring is never compressed. Alternatively, it can be ensured that the spring is only extended by using the constraint $r \leq (OM - l_0)$. The constraint $0 \leq \alpha < 2\pi$ is included to avoid the periodicity of the angle α.

In order to test the capabilities of the proposed optimization method, a numerical test case is taken into consideration. The mechanism parameters are reported in Table 1. A cycloidal motion trajectory, designed to achieve a full rotation of the mechanism in $T = 0.5$ s, is considered. The optimization procedure, which in this case does not include k among the decision variables, brings to the following optimal parameters: $r = 0.278$ m, $\alpha = 3.0962$ rad. Figure 2 shows the motor torque needed to follow the prescribed trajectory with and without the inclusion of the optimized spring design, as well as the equivalent torque exerted by the spring. Without using the spring, the peak torque requirement is equal to 0.7401 Nm, while the inclusion of the spring allows to reduce such value to just 0.3915 Nm, thus reducing the

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peak torque by 47.1%. This result shows that, despite the conceptual and technical simplicity of the proposed approach, the possible motor size reduction is far from being negligible.

Table 1 Mechanism prototype parameters

Symbol	Value
J	$4 \times 10^{-3}\ kgm^2$
μ	$0.008\ Nms/rad$
k	$88.5\ N/m$
OM	$0.08\ m$

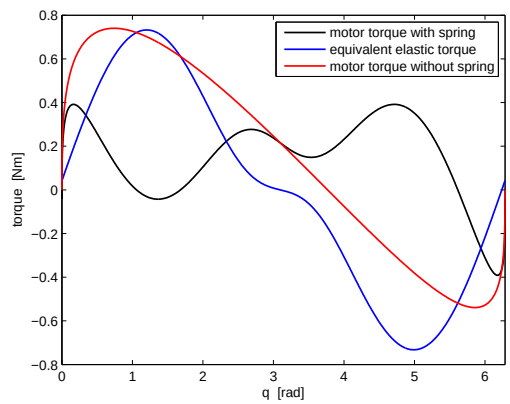

Fig. 2 Motor torque with and without the spring: cycloidal motion

Another test has been performed by taking into consideration another trajectory, namely a 5th order polynomial with null initial and final acceleration, with the same overall execution time. In this case the peak value of the motor torque is reduced from 0.6724 Nm to 0.4095 Nm, therefore the inclusion of the spring can reduce the peak torque requirement by 39.11%. The corresponding optimal spring placement is: $r = 0.27$ m, $\alpha = 3.0987$ rad. The torque requirements with and without the spring, as well as the designed spring equivalent torque are shown in Fig.3.

4 Experimental results

In order to test the capabilities and the effectiveness of the proposed design approach, a prototype of the mechanism, shown in Fig.4, has been built. The testbench comprises a single aluminum link, a brushless motor and a rubber band in lieu of the

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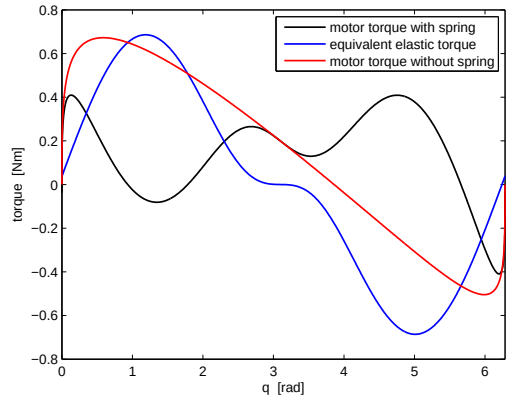

Fig. 3 Motor torque with and without the spring: 5th degree polynomial motion

spring. The rubber band is represented by its average stiffness, equal to 88.5 N/m and the free length $l_0 = 0.19$ m. The overall link length is equal to 0.38 m, and the distance between the motor axis and the location of the movable end of the rubber band is equal to 0.08 m. The resulting overall inertia about the motor axis is equal to $3.88 \times 10^{-3}\ kgm^2$.

Fig. 4 Experimental test: mechanism prototype

A first experimental test has been performed by driving the actuator along a cycloidal trajectory, designed to provide a full rotation of the mechanism in 0.5 s. Figure 5 shows the estimated torque provided by the actuator, with and without the additional elastic elements. The location of the fixed end of the spring has been set as suggested by the results of the optimization procedures reported in the previous section.

The motor torque has been estimated on the basis of the current absorption data collected by the PLC which drives the motor, by multiplying it by the torque constant listed in the motor's datasheet. This method has been chosen due to the unavailability of a torque transducer or a direct measurement of the windings currents.

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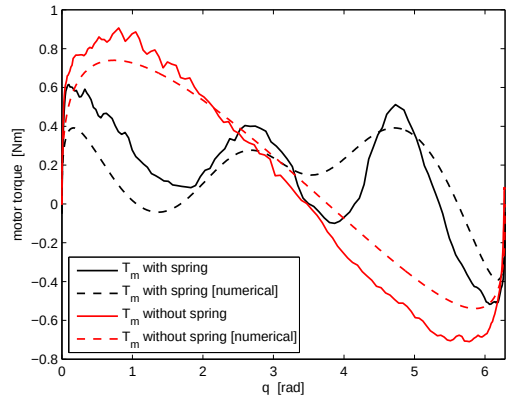

Fig. 5 Experimental test: motor torque with and without the spring, cycloidal motion

The data shown in fig. 5 show that the inclusion of the spring allows to reduce the peak torque requirement from 0.9067 Nm to 0.6156 Nm, with a resulting 32.1% reduction. According to the numerical model, the predicted reduction was equal to 47.1%: the difference between the two improvement figures is due to the reduced accuracy of the numerical model, as well as the limited accuracy of the torque estimation during the experimental tests. The torque profile show in Fig. 5, for example, highlights an evident torque ripple, which is not predicted by the dynamic model of eq. 1. However, the experimental test confirm that, even without the aid of an extremely accurate dynamic model, a noticeable improvement can be achieved.

A similar test has been performed using the fifth-order polynomial trajectory, again performing the full rotation of the crank in 0.5 s. The resulting estimated torque is reported in Fig. 6. The analysis of the estimated torque show a 24.9% reduction of the peak torque as the effect of adding the elastic element, being the maximum torque equal to 0.8861 Nm and to 0.6654 Nm, without and with the rubber band, respectively. Again, the actual improvement is smaller than the predicted one, but still far from being negligible.

5 Conclusion

In this paper a method to reduce the motor size in articulated mechanism has been presented. The proposed method, which is applied to a single-link mechanism, allows to evaluate the optimal spring characteristics and the optimal spring placement with the aim of reducing the peak torque requirement. The optimization procedure also takes into account the choice of the joint trajectory, which is shown to affect the result of the optimization procedure. A significant improvement is highlighted both using numerical evaluation and experimental tests performed on a mechanism prototype.

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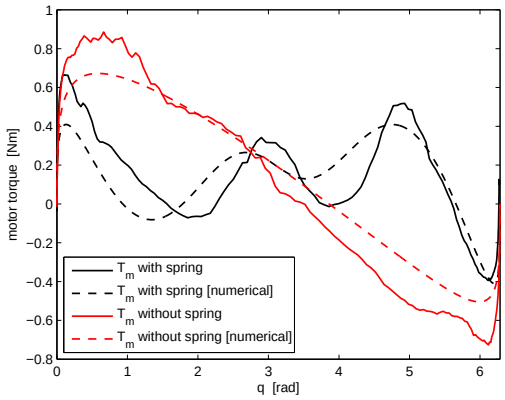

Fig. 6 Experimental test: motor torque with and without the spring, fifth order polynomial trajectory

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