# Energy saving in redundant robotic cells: optimal trajectory planning

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## Abstract

This work proposes a method to increase the energy efficiency of robotic cells by exploiting redundancy. The test case proposed for validation is a system typically used for fast pick and placements operations: a three degrees of freedom SCARA robot integrated with a separate linear unit moving the workpiece that provides an additional redundant degree of freedom. The design of the motion profile of the SCARA robot is based on splines interpolants defined by a sequence of via-points, while the motion of the auxiliary linear axis is described by a piecewise trajectory, so that both motions achieve acceleration continuity. The results show that a careful concurrent design of the trajectories of the robot and of the auxiliary axis can provide a significant reduction of the required energy.

## 1 Introduction

The production cost of manufacturing operations is heavily affected by the cost of the electric energy needed to operate the robot cells which provide most of the work, especially when high volume and precision are the key elements of a cost-effective productive environment. The importance of the topic, and its relevance to both the scientific community and the industry is testified by a flourishing literature [1], as well as by the directions set by the European Union policy, which aims at reducing the whole energy consumption up to 30% by 2030. Several theoretical [2] and experimental investigations [3] have shown that the potential energy saving brought by a wise operation of robotic systems, as well as the use of regenerative drives, can lead to noticeable energetic demand reduction, which can reach up to 30%.

The work [1] lists several areas of intervention for achieving a greener robot operation, such as lightweight robot design [4], energy recovery and storage, motion planning [5] and robot architecture selection [6]. The influence of motion planning on the consumption of electric-driven mechatronic devices has been investigated since the late Seventies. In the meanwhile it has been shown that simple analytical models can be efficiently used to evaluate and optimize the power consumption of a mechatronic device [7, 8]. Such models can be used for the analytical design of energetically optimal motion profiles for simple applications, while numerical optimization methods are the common choice for more complicated systems [9].

Another tool that can lead to a significant energetic improvement is the exploitation of kinematic redundancy, i.e. of extra degrees of freedom by selecting the energy-optimal solution of the inverse kinematic problem among the infinite ones [10].

This work provides a novel solution for this problem by discussing the optimization of the energy consumption of a redundant robotic system, comprising a three degrees-of-freedom SCARA robot and an additional linear unit that is used to move the workpiece. An energy optimal trajectory planning algorithm is then computed, by designing piecewise motion profiles, on the basis of an electromechanical model of the whole robotic cell. The energy saving is then evaluated against a similar planning method applied to the non-redundant configuration obtained simply by restricting the motion of the linear unit.

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## 2 Trajectory planning and optimization

The proposed trajectory planning method is based on the use of via-point to specify the path of the end-effector of the robot, by forcing the end-effector trajectory to interpolate them. Via-points, which are here specified as locations in a three-dimensional Cartesian space, can be then mapped to the joint space by the inverse kinematic map, thus transforming to this space the trajectory planning problem. The latter is, in the proposed method, cast as an interpolation problem. In particular, a trajectory defined by a set of via-points can be described by a set of splines function of different degrees, arranged so that continuity up to the second derivative, i.e. acceleration, can be achieved. Among the several methods that reach such goal, the so-called '4-3-4' method has been chosen. The '4-3-4' method can be implemented [11] to include kinematic limits in the trajectory design, i.e. maximum values of speed, acceleration and jerk can be set to ensure the feasibility of the motion. The use of this method is also very convenient when used within a numerical optimization routine, since it provides a simple parametrization of the trajectory, ensuring that each implementation of the motion profiles is described simply by the duration of the time intervals over which the robot moves between two consecutive via-points. In other words, if $N$ via-points are specified, the trajectory of the robot is uniquely defined by $N-1$ time intervals.

Since the target of the work is the implementation of a method to reduce the energy consumption of the robotic cell in a pick-and-place operation, the dynamic model that will be used to estimate the electric energy consumption will be briefly introduced. The test-case under consideration here is a robotic cell which comprises of a three d.o.f. SCARA robot and a sliding table, which is actuated by a linear brushless motor. A schematic view of the layout of the robotic cell is shown in Fig. 1.

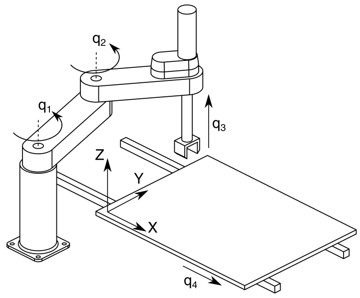


Figure 1: Layout of the robotic cell

The dynamic model of the SCARA robot can be described by using the Lagrangian formalism, leading to the usual formulation:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{f}_v\dot{\mathbf{q}} + \mathbf{F}_c\,\mathrm{sign}\,(\dot{\mathbf{q}}) = \boldsymbol{\tau}_m \quad (1)$$

which involves the vector of joint coordinates $\mathbf{q} = [q_1, q_2, q_3]^T$, the diagonal matrix $\mathbf{f}_v$ of viscous friction coefficients and the diagonal matrix of Coulomb friction forces $\mathbf{F}_c$. Motor torques at the joint are represented by the three components of vector $\boldsymbol{\tau}_m$. $\mathbf{M}(\mathbf{q})$ is the configuration-dependent mass matrix, while $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ accounts for the centrifugal effects. The electromechanical model of the motors that drive the SCARA robot can be introduced into Eq.(1), by recalling that the motor currents and the motor torques can be related by the diagonal matrix of the individual motor torque constants, $\mathbf{k}_t$, as:

$$\boldsymbol{\tau}_m(t) = \mathbf{k}_t\mathbf{I}(t) \quad (2)$$

The voltage drop across the motors can then be described by the armature model, which collects the contributions for all the joints of the robot:

$$\mathbf{V}(t) = \mathbf{R}\mathbf{I}(t) + \mathbf{k}_b\dot{\mathbf{q}}_m(t) \quad (3)$$

being $\dot{\mathbf{q}}_m$ the vector of the motor velocities and $\mathbf{k}_b$ the diagonal matrix of the motors' back-emf constants. The instantaneous power drawn by the robot is then simply expressed by the current-voltage product, so that the overall energy consumption for the time interval $[t_a, t_b]$ is found by computing the time integral:

$$E_{SCARA} = \int_{t_a}^{t_b} \mathbf{V}^T(t)\mathbf{I}(t)dt \quad (4)$$

This model can be implemented within the trajectory optimization routine, which will include a numerical integration of Eq.(4), following the approach commonly used in works such as [9].

The estimation of the energy needed to drive the fourth axis of the robotic cell, i.e. the sliding table, can be performed in a more computationally efficient way through its analytic formulation, which can be found if an analytic formulation rather than through a numerical integration. As a meaningful example, the so-called

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'symmetric double-S profile' is chosen in this work. Such a profile is defined as follows:

$$\begin{aligned}
&\dot{q}_4(t) = v_0\left(1 - cos\left(w_1 t\right)\right)/2, && t \in [0, t_1)\\
&\dot{q}_4(t) = v_0, && t \in [t_1, T - t_1]\\
&\dot{q}_4(t) = v_0\left(1 + cos\left(w_1 t\right)\right)/2, && t \in (T - t_1, T]\\
&\text{with: } v_0 = H/\left(T - t_1\right) &&
\end{aligned} \tag{5}$$

According to Eq.(5) the linear unit will move at constant speed during the intermediate phase of the motion, which is preceded and followed by two sinusoidal acceleration and deceleration phases, respectively, whose duration is $t_1$ for both. The overall displacement $H$ is performed over the time duration $T$.

The dynamic model of the linear unit is needed to follow a similar procedure to the one used to describe the energy consumption of the SCARA robot. If the moving mass of the linear unit is $m$ and its speed is $\dot{q}_4(t)$, the dynamic model of the linear unit is decoupled from the one of the SCARA robot, and is:

$$m\ddot{q}_4(t) + f_v\dot{q}_4(t) + F_c\,\mathrm{sign}\left(\dot{q}_4(t)\right) = F_m(t) \tag{6}$$

Eq.(6) relates the force $F_m(t)$ exerted by the brushless linear motor that drives the linear unit with the inertial force, the viscous force (through the friction coefficient $f_v$) and the Coulomb friction force $F_c$. The electric power drawn by the linear motor can be represented as the voltage-current product. By exploiting the same model used for the SCARA robot in Eq.(2) and (3), using the linear brushless motor resistance $R$, the torque constant $k_t$ and the back-emf constant $k_t$, the instantaneous electric power is:

$$\begin{aligned}
W_{LU}(t) = &\frac{R}{k_t^2}\left(m^2\ddot{q}_4^2(t) + f_v\dot{q}_4^2(t) + f_c^2\right)\\
&+ \frac{R}{k_t^2}\left(2mf_v\dot{q}_4(t)\ddot{q}_4(t) + 2f_vf_c\dot{q}_4(t)\right)\\
&+ \frac{k_b}{k_t}\left(m\dot{q}_4(t)\ddot{q}_4(t) + f_v\dot{q}_4^2(t) + f_c\dot{q}_4(t)\right)
\end{aligned} \tag{7}$$

The energy associated with the execution of the motion profile described by Eq.(5) can be evaluated by computing the time integral of the electric power in Eq.(7) over the time interval $[0, T]$. The piecewise integration of the speed and acceleration profiles shows that, for the prescribed motion, the following holds:

$$\int_0^T \ddot{q}_4(t)dt = 0 \tag{8}$$

$$\int_0^T \dot{q}_4^2(t)dt = v_0^2\left(T - \frac{5}{4}t_1\right) \tag{9}$$

$$\int_0^T \ddot{q}_4^2(t)dt = 0 \tag{10}$$

$$\int_0^T \dot{q}_4(t)dt = H \tag{11}$$

$$\int_0^T \dot{q}_4(t)\ddot{q}_4(t)dt = 0 \tag{12}$$

$$\int_0^T \ddot{q}_4^2(t)dt = \frac{\pi^2 v_0^2}{4t_1} \tag{13}$$

Collecting all the terms in Eq.(8–13) leads to the evaluation of the energy required by the linear unit to perform the whole rest-to-rest motion, whose overall displacement is $H$, as:

$$\begin{aligned}
E_{LU} = &\frac{R}{k_t^2}\left(m\frac{v_0^2\pi^2}{4t_1} + f_v v_0^2\left(T - \frac{5}{4}t_1\right)\right)\\
&+ \frac{R}{k_t^2}\left(d_c^2 T + 2f_vf_cH\right)\\
&+ \frac{k_b}{k_t}\left(f_v v_0^2\left(T - \frac{5}{4}t_1\right) + f_cH\right)
\end{aligned} \tag{14}$$

The latter can be expressed in terms of just the total execution time $T$ and the acceleration (and deceleration) time $t_1$. The energy consumption of the whole robotic cell can be evaluated by summing the two energy contributions using Eq.(4) and Eq.(14). The design of the trajectory is set-up in this work as the following optimization problem:

$$\begin{aligned}
&\underset{[\mathbf{T}, t_1]}{\text{minimize}}\ E_{SCARA} + E_{LU}\\
&\text{with:} \qquad \mathbf{T} \in \Re^{N-1}, t_1 \in (0, T_1/2]\\
&\text{subject to: } \textstyle\sum_{i=1}^{N-1} T_i = T\\
&\qquad\qquad \text{bounded } |\dot{\mathbf{q}}|, |\ddot{\mathbf{q}}|, |\dddot{\mathbf{q}}|
\end{aligned} \tag{15}$$

The optimization problem of Eq.(15) states that a minimum energy solution is sought by setting the correct values of the $N-1$ time intervals $T_i$ and the acceleration time $t_i$ of the linear unit, so that the overall execution time is set to $T$, with the inclusion of speed, acceleration and jerk bounds. The task is specified by $N$ via-points that are usually defined, for the operator's convenience, in the operative space, then the computation of the optimization problem of Eq.(15) is performed after their transformation to the joint space.

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The energetic performance of the proposed method is then measured in comparison with a non-redundant solution, i.e. the one obtained without using the linear unit, i.e. by forcing $\dot{q}_4(t) = 0$. The benchmark problem taken into consideration is set-up by 6 via-points defined in a reference frame that is located on the sliding table. Via-points are chosen to produce a pick and place task that involves the motion of all the axis of the cell. In the redundant case the motion along the $X$ axis is, by choice, performed only by the linear unit, while in the non-redundant case the same motion is performed only by the SCARA robot.

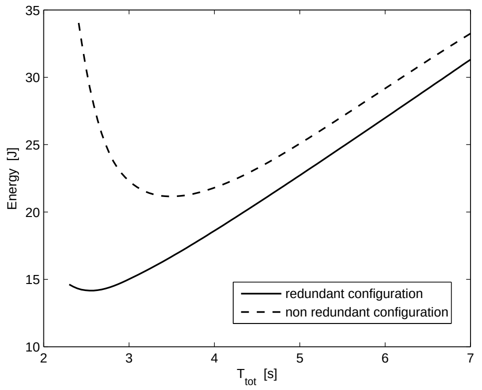


Figure 2: Energy consumption vs. total execution time: comparison between redundant and non-redundant configuration

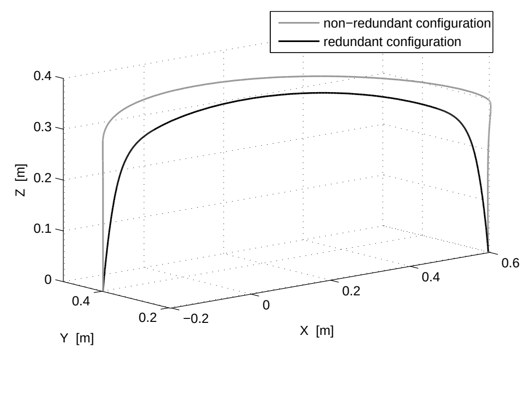


Figure 3: End-effector paths for the optimal solution: non-redundant configuration vs redundant configuration

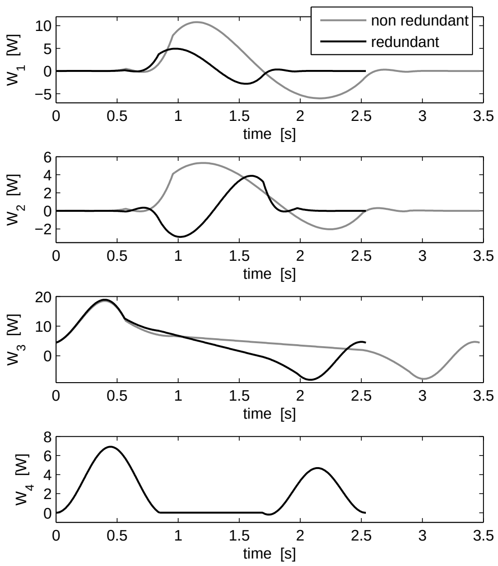


Figure 4: Absorbed electric power for the optimal solutions: non-redundant configuration vs redundant configuration

The optimization problem of Eq.(15) has been solved iteratively to design the energy optimal motion profiles with total execution times ranging from $T = 7$ s to $T = 2$ s, stopping the procedure when a speed, acceleration or jerk constraint violation was detected, indicating that the minimum execution time for the task has been found. The procedure has been repeated for the non-redundant configuration as well, and the results are compared in Fig. 2. Such results indicate that the energy consumption in the redundant case is lower than the one achieved without the use of the additional linear unit regardless of the total execution time of the task. For the redundant configuration the lowest energy usage is found for a total execution time equal to $T = 2.547$ s, for which 14.62 J are needed, while in the non-redundant cases the two figures are, respectively, equal to $T = 3.468$ s and 21.16 J. The resulting paths of the end-effector are shown in Fig. 3, which provides a comparison for the two aforementioned optimal solutions. The redundant solution results in a slight distortion of the path, which can be compensated, if needed, by imposing some rest time to the linear unit at the initial and final phases of the task.

The instantaneous electric power required by each actuator is plotted in Fig. 4. The analysis of these data shows that the redundant configu-

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ration can compensate for the additional power consumption introduced by the operation of the fourth axis of the robotic cell by reducing the power required by the actuators that move the first and the second joint of the robot. Moreover, an additional improvement is that also the total execution time of the trajectory can be made significantly lower for a given maximum allowed energy expenditure, with obvious advantages on the productivity of the robotic cell.

# 3 Conclusion

This works proposes a solution to the problem of designing energy optimal tasks for robotic cells by exploiting kinematic redundancy as a tool for speeding up and reducing the overall energy consumption in pick and place operations. The test case under consideration is a robotic cell designed to include a three degrees of freedom SCARA robot and a separate linear unit that is used to move the workpiece. The design of the motion profiles is based on the use of piecewise functions, using splines for the robot and double-S velocity profiles for the linear unit. An optimization-based trajectory planning algorithm is then used together with an inverse dynamic model of the robotic cell. Numerical results indicate that the proposed method allows for a significant reduction of the total energy required to perform the prescribed task, as well as a significant speed-up of the robotic cell.

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