Spline-based energy-optimal trajectory planning for functionally redundant robots

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***Abstract*—The design of optimal trajectories for automatic machines is an effective tool to reduce their energy consumption. This work investigates the topic by proposing a method tailored for functionally redundant robots, i.e. the ones which have more degrees of freedom than the ones required by the task. The test case under consideration is a serial 3R robot used for a positioning task in a planar space, described by a sequence of via-points in the operative space. Speed limits and smoothness constraints, in terms of speed and jerk limitations are taken into account to ensure trajectory feasibility. The method finds the optimal time intervals between two consecutive via-points, as well as the optimal robot configurations at the via-points (i.e. the optimal solution of the inverse kinematic problem among the infinite ones). The results show the capability of the method in producing energy-efficient motion profile, and the improved results over the optimization of just the time intervals.**

Keywords—Robot, Trajectory planning; Energy efficiency; Functional redundancy; Spline.

I. INTRODUCTION

A. *State of the art in energy reduction through motion planning*

A significant part of the manufacturing costs is nowadays due to the price of the energy needed to operate robot and automatic machines employed in the process. Therefore, any improvement in this area can be economically beneficial at any level. The attention to this topic is testified not only by a flourishing scientific literature, but also from the directions set by the European Union policy [1] which aims at reducing the primary energy consumption of 30% by 2030. For this reason, the use of technologies and strategies for the optimization of the energetic costs of robots is of major importance. For example, papers [2] and [3] showcase the application of several strategies for achieving power savings up to 40%, by adopting smart solutions for motion planning, energy sharing and intelligent brake managing.

The availability of such technological advancements suggests the development of novel robot operation planning strategies specifically aimed at minimizing the energy consumption, such as trajectory planning algorithms. The publications of the recent patents [4, 5] that focus on motion planning as a tool for improving the energy efficiency proves that this topic is perceived as a fundamental one by robot practitioners and manufacturers as well. The scientific literature on the topic is wide and is recently flourishing, as highlighted in the recent review [6]. The problem of analyzing the impact of the choice of the trajectory of machines operated by electric motors has been investigated since the late Seventies, such as in [7], in which a framework for estimating the energy consumption of DC motors is introduced. Further developments of the same technique, i.e. the integration of the energy for reproducing a trajectory described by a simple analytic formulation, can be found in [8] and [9]. Such works belong to the class of indirect methods, which translate an optimization problem in a parameter optimization one, whose solution if found numerically. In contrast, the work [10] introduces a method that enables a straightforward computation of the energetic cost for rest-to-rest motion of constant inertia systems, allowing to perform the analytical optimization without resorting to numerical integration or iterative optimization procedures.

Other proposed trajectory optimization methods are indirect ones, being based on the use of variational calculus, and in particular on the solution of Euler's equations, such as [11,12]. The use of variational calculus is however a method that can be of lesser effectiveness for multiple degrees of freedom and for complex problems that include several constraints (such as the ones that are considered in this work) since their numerical solution is often incompatible with larger number of state variables problems [13].

The possibility of improving energy efficiency is even greater in the presence of redundant manipulation systems, where infinite configurations leads to the same end-effector path and therefore the presence of additional dofs can be used to enhance the performance in some sense. For example, in [14] energy reduction is obtained through minimum energy control, for a redundant linear manipulator.

B. *Aims and scope of the work*

The advantages of redundancy can be exploited even in robots that are not intrinsically redundant in the execution of some particular tasks whose number of required degrees of freedom (dofs) is smaller than the number of dofs of the robot. This is the so-called functional redundancy, and this work is focused on the definition and the solution of energy-optimal trajectories for functionally redundant robots. A robot is said to be functionally redundant when the dimension of the operational space is greater than the dimension of the task space

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[15]. Functional redundancy has been proved to be effective to improve a secondary requirement, e.g. to improve accuracy as testified in [16], while ensuring exact positioning of the end-effector. This paper exploits redundancy to minimize the electric energy consumption of a robot, by assuming regenerative electric motors sharing over a common DC bus. Splines are used to interpolate a set of prescribed via-point defined in the operational space, to ensure continuity of speed, acceleration and jerk. The proposed method solves the inverse kinematic problem by selecting the set of solutions minimizing the electric energy consumption computed through the dynamic model of the robot and of the electric actuators.

II. Spline-based Trajectory Planning

The computation of energy-efficient trajectories proposed in this work is based on a well-established framework that describes the task as a sequence of N via-points defined in the operational space. Hence the resulting path of the end-effector will intersect such points [17]. Each via-point is mapped to the joint space by using a suitable kinematic inversion algorithm. After that, the robot trajectory can be planned by simply applying a suitable interpolation method, such as spline functions. The method used in this work is based on the use of the so-called "4-4-5" spline interpolation method [18]. Such a method produces the trajectory as a sequence of fourth-degree polynomials defined within two consecutive via-points in the joint space, plus a fifth-degree polynomial in the last segment. This method has the desirable property of producing trajectories that are continuous up to the fourth-order derivative, i.e. leads to continuous jerk. Jerk continuity, which is often referred to as smoothness of the trajectory, is a commonly sought property of a trajectory [17], together with jerk boundedness. Indeed, such properties ensure less generation of motion-induced vibration, and hence better control of the robot and a more accurate motion tracking. Another advantage offered by "4-4-5" trajectories lies in the reduced set of parameters needed to synthesize them, since each the motion is just described by the positions of N via-points and the time duration of each segment of the trajectory. A more detailed formulation can be found in [18]. Here a quick presentation is provided. To avoid confusing formulations, all the formulas are reported for a single axis of the robot, since the extension to the multiple axes case is straightforward.

Let us consider a trajectory defined through N via-points $\boldsymbol{Q} = [q_{i,1} \dots q_{i,N}]$ in the joint space, which are supposed to be computed by an inverse kinematic algorithm from their equivalents poses in the operative space. If fourth-degree polynomial functions are used to describe the segments between two consecutive via-points, except for the last one, the trajectory between two adjacent via-points $q_{i,k}$ and $q_{i,k+1}$ ($1 \leq k \leq N-2$) can be written as:

$$F_k(t) = B_{k,1} + B_{k,2}t + B_{k,3}t^2 + B_{k,4}t^3 + B_{k,5}t^4 \tag{1}$$

in which $F_k(t)$ represents the planned position of a single joint of the robot for the k-th segment of the trajectory. The values of the polynomial coefficients $B_{k,n}$ are defined to ensure continuity of velocity, acceleration and jerk between two consecutive segments, i.e. at each via-point. Let T_k be the duration of each interval; then such continuity conditions are represented using the following constraint:

$$\begin{bmatrix} B_{k,1} \\ B_{k,2} \\ B_{k,3} \\ B_{k,4} \\ B_{k,5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 \\ -\frac{4}{T_k^3} & -\frac{3}{T_k^2} & -\frac{1}{T_k} & \frac{4}{T_k^3} & -\frac{1}{T_k^2} \\ \frac{3}{T_k^4} & \frac{2}{T_k^3} & \frac{1}{2T_k^2} & -\frac{3}{T_k^4} & \frac{1}{T_k^3} \end{bmatrix} \begin{bmatrix} q_{i,k} \\ v_{i,k} \\ a_{i,k} \\ q_{i,k+1} \\ v_{i,k+1} \end{bmatrix} \tag{2}$$

in which v_k and a_k are the speed and the acceleration at the k-th via-point, respectively. The time history of the planned position for the last via-point is instead described by the fifth-degree polynomial function:

$$F_{N-1}(t) = B_{N-1,1} + B_{N-1,2}t + B_{N-1,3}t^2 + B_{N-1,4}t^3 + B_{N-1,5}t^4 + B_{N-1,6}t^5 \tag{3}$$

The kinematic constraints that impose continuity between the last and the next-to last segment can be translated into the following matrix relationship:

$$\begin{bmatrix} B_{N-1,1} \\ B_{N-1,2} \\ B_{N-1,3} \\ B_{N-1,4} \\ B_{N-1,5} \\ B_{N-1,6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 \\ -\frac{10}{T_{N-1}^3} & -\frac{6}{T_{N-1}^2} & -\frac{3}{2T_{N-1}^2} & \frac{10}{T_{N-1}^3} & -\frac{4}{T_{N-1}^2} & \frac{10}{2T_{N-1}} \\ \frac{15}{T_{N-1}^4} & \frac{8}{T_{N-1}^3} & -\frac{3}{T_{N-1}^2} & -\frac{15}{T_{N-1}^4} & \frac{7}{T_{N-1}^3} & -\frac{1}{T_{N-1}^2} \\ -\frac{6}{T_{N-1}^5} & -\frac{3}{T_{N-1}^4} & -\frac{1}{2T_{N-1}^3} & \frac{6}{T_{N-1}^5} & -\frac{3}{T_{N-1}^4} & \frac{1}{2T_{N-1}^3} \end{bmatrix} \begin{bmatrix} q_{i,k} \\ v_{i,k} \\ a_{i,k} \\ q_{i,k+1} \\ v_{i,k+1} \\ a_{i,k+1} \end{bmatrix} \tag{4}$$

If taken separately, (2) and (4) can be solved only if all the velocities v_k and accelerations a_k are defined, which is not the case here since just continuity has been required. A further development of the formulation is therefore needed [18], by transforming the resulting system in the form:

$$\boldsymbol{Ad} = \boldsymbol{h} \tag{5}$$

Vector $\boldsymbol{d}$ collects the unknown via-point velocities and accelerations:

$$\boldsymbol{d} = [v_2 \quad a_2 \quad v_3 \quad a_3 \quad \cdots \quad v_{N-1} \quad a_{N-1}] \tag{6}$$

while $\boldsymbol{h}$ is the vector of coefficients h_k, that are linear combinations of the time intervals T_k and of the via-point positions, as well as of the initial and final velocities and accelerations:

$$\boldsymbol{h} = [h_1 \quad h_2 \quad h_3 \quad \cdots \quad h_{2N-5} \quad h_{2N-4}] \tag{7}$$

$\boldsymbol{A}$ is a $(2N-4) \times (2N-4)$ matrix which depends just on the time intervals T_k. The procedure outlined here shows that, for a choice of the N via-points and of the initial and final

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velocities and acceleration, each trajectory is completely defined by the choice of the $N-1$ time intervals T_k. This choice allows to solve for $\boldsymbol{d}$ using Eq. (5), i.e. the unknown velocities and accelerations at via-points are made available. From the latter, the polynomial coefficients $B_{k,n}$ are found and then used to compute the actual motion profiles and verify kinematic limits violations. In contrast, if the time intervals and some of the via-point positions are not prescribed, and hence are treated as variables, the resulting system is nonlinear and non-separable.

In this work, these variables are computed to minimize the energy consumption by means of an optimization problem. The result is an energy-optimal trajectory that exploits arbitrary values of time durations, speeds and accelerations at the via-points and the position of the redundant dofs, while satisfying some design constraints.

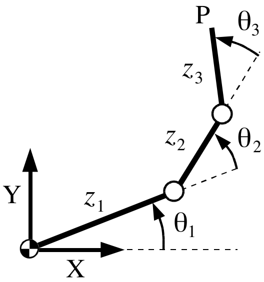

Fig. 1. 3R serial manipulator: kinematic model

III. Energy consumption estimation

In this section the analytical model used for estimating the energy consumption of a robotic manipulator driven by electric motors is developed. The formulation used here can be adopted to any arbitrary robot, but specific reference is made to the planar 3R robot lying in the horizontal plane, that is chosen as the test-case. The kinematic structure of the manipulator is made by three links and three revolute joints arranged as a serial structure. The robot kinematic model is shown in Fig. 1. The motion of the robot link can be described by the motor joint positions collected in $\boldsymbol{q}=[q_1,q_2,q_3]^T$, that are related to the link position through the gear ratios of three gearboxes. The manipulator is functionally redundant if the task is specified just in terms cartesian coordinates of the end-effector position $\boldsymbol{P}$, while no orientations of the end-effector are specified.

The dynamics of the manipulator, which is needed to solve the inverse dynamic problem, can be defined using the Lagrangian formalism [15], leading to the usual formulation:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})+\boldsymbol{F}_v\dot{\boldsymbol{q}}=\boldsymbol{T}_m \tag{8}$$

which includes the effect of viscous friction through the diagonal matrix $\boldsymbol{F}_v$ collecting the friction coefficients $\boldsymbol{f}_v$, the vector of motor torques $\boldsymbol{T}_m$. $\boldsymbol{M}(\boldsymbol{q})$ is the configuration-dependent mass matrix and $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ collects centrifugal effects. The electromechanical model of the motor can be introduced in (8) as well, recalling that for an electric motor the exerted torque is proportional to the current drawn by the motor. For each axis, the relation is:

$$T_{m,i}(t)=k_{t,i}I_i(t) \tag{9}$$

being $k_{t,i}$ the motor torque constant.

TABLE I. Parameters of the manipulator

	Link/joint 1	*Link/joint 2*	*Link/joint 3*
Link length $[\boldsymbol{m}]$	1	0.5	0.5
Link mass $[\boldsymbol{kg}]$	5	2.5	2.5
Link inertia $[\boldsymbol{kgm^2}]$	0.4167	0.0521	0.0521
Gear ratio	1/5	1/5	1/5
$\boldsymbol{f_v}$ $[\boldsymbol{Nm\,s/rad}]$	1e-3	1e-3	1e-3
$\boldsymbol{k_t}$ $[\boldsymbol{Nm}/\mathrm{A}]$	0.65	0.65	0.65
$\boldsymbol{k_b}$ $[\boldsymbol{V\,s/rad}]$	0.65	0.65	0.65
$\boldsymbol{R}$ $[\Omega]$	2	3.3	4

The voltage drop across the i-th motor is in turn described by the armature model:

$$V_i(t)=R_iI_i(t)+k_{b,i}\dot{q}_i(t) \tag{10}$$

in which R_i is the resistance of the motor windings and $k_{b,i}$ is the back-emf constant (the effect of the inductance can be neglected). The total energy consumption over a time interval can be therefore estimated as the sum of the time integral of the electric power absorbed by each axis:

$$E=\sum_{i=1}^{3}\int_{t_a}^{t_b}V_i(t)I_i(t)dt \tag{11}$$

Indeed, the overall power consumption for a multi-dof system with energy sharing on a common bus and regenerative drives can be evaluated simply by summing each individual power consumption (with a minor approximation due to the high efficiency of regeneration). In this case, each actuator can act both as a motor or as a generator, since the current generated by a single actuator can be used by another one sharing the same bus. Such a feature is commonly found in several applications, given the potential energy efficiency improvement over less elaborated system that dissipate any regenerated current on a braking resistor [19]. This electric consumption model has been used in several works, such as [20,21], and its capability of accurately predicting the energy consumption of an industrial application has been reported, among others, in [8].

IV. Trajectory optmization

A. The proposed method

The novel method for energy-optimal trajectories is aimed at exploiting the functional redundancy sported by the three-dof robot when executing a positioning task in a planar workspace. The extra dof, that leads to ∞^1 possible solutions of the inverse kinematic problem, i.e. to infinite robot configurations that

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locate the end effector at the desired via-point, will be therefore used to enhance energy efficiency. Trajectories are designed in the joint space to include explicitly constraints on the maximum absolute joint speed, acceleration and jerk. Such constraints have the purpose of complying with both the robot specifications and with the desired smoothness.

The proposed method includes within the optimization variables both the durations of the N-1 time intervals and the N position of joint 1 at the N via-points. Once q_1 is set, the inverse kinematic problem is well-posed and can be solved for q_2 and q_3. The following optimization problem can therefore be stated:

$$\min_{\boldsymbol{T},\boldsymbol{q}_{1,k}}\{E\} \quad with\ \boldsymbol{T}\in\mathbb{R}^{N-1},\ \boldsymbol{q}_{1,k}\in\mathbb{R}^{N}$$
$$subject\ to: \tag{12}$$
$$\begin{cases} |\dot{q}_i| \le v_{i,max} \\ |\ddot{q}_i| \le a_{i,max} \\ |\dddot{q}_i| \le j_{i,max} \\ \sum_{k=1}^{N-1} T_k = T_{tot} \\ \boldsymbol{Ad}-\boldsymbol{h}=\boldsymbol{0} \\ \Phi(\mathbf{q})=\mathbf{0} \end{cases}$$

$\Phi(\boldsymbol{q})$ in Eq. (12) denotes the kinematic constraint equations. Equation (5) is treated as a non-linear equality constraint. Additionally, the motion duration is set to be exactly T_{tot}. Constraints on the maximum speeds, accelerations and jerks along the whole trajectory are set through $v_{i,max}$, $a_{i,max}$, $j_{i,max}$. This formulation can be extended to problems with higher degree of redundancy, provided that a convenient parametrization of the ∞^n solutions for the inverse kinematic problem is used. If needed, a lower bound on manipulability can be set to avoid operating in proximity of a singular configuration.

Even if optimization problem in not convex, a wise selection of the initial guess gets rid of this issue and boosts the achievement of significant energy reductions. As for the time durations is concerned, the popular cord length or centripetal distributions methods, that are often proposed in the literature [17], can be adopted. As for the initial guess of the joint positions at the via-points, they might be computed through heuristic approaches based on kinematic or dynamic performance indexes.

B. The benchmark methods

Two benchmark methods are used here to evaluate the results obtained by the proposed planning method. Each benchmark relies on a "performance index" $w_i(\boldsymbol{q})$ to choose the inverse kinematic solution among the infinite possible ones. In accordance to a wide literature, a first benchmark is established by choosing the pose of the robot that maximizes the popular manipulability index [22], i.e. that minimizes its reciprocal. The manipulability index measures the volume of the velocity manipulator ellipsoid. Therefore, large values of such an index ensure that the end-effector can produce large velocities. In the first benchmark $w_1(\boldsymbol{q})$ is therefore defined through the Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$ as follows:

$$w_1(\boldsymbol{q}) = 1\Big/\sqrt{\det(\boldsymbol{J}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q}))} \tag{13}$$

The second benchmark selects the sequence of the robot poses at each via-point by minimizing the overall displacements of the three joints. Hence the index is defined as:

$$w_2(\boldsymbol{q}) = \sum_{k=1}^{N-1}\sum_{i=1}^{3}\left|q_{i,k+1}-q_{i,k}\right| \tag{14}$$

In practice, such benchmarks are heuristic approaches that lead to reasonable solutions of the inverse kinematic problem. The resulting planning approaches, however, do not embed the inverse kinematic problem within the optimization problem and just include the time intervals as the optimization variables. In contrast, the set of joint positions locating the end-effector in the desired via-points are chosen a-priori as those optimizing the two performance indexes for each via-point. Then, the energy optimization problem is aimed at finding the optimal set of time intervals $\boldsymbol{T}$ that ensure energy minimization within the prescribed constraints. The optimized trajectory will be their jerk-continuous interpolation that achieves energy optimality, according to the following problem:

$$\min_{\boldsymbol{T}}\{E\} \quad with\ \boldsymbol{T}\in\mathbb{R}^{N-1}$$
$$subject\ to: \tag{15}$$
$$\begin{cases} |\dot{q}_i| \le v_{i,max} \\ |\ddot{q}_i| \le a_{i,max} \\ |\dddot{q}_i| \le j_{i,max} \\ \sum_{k=1}^{N-1} T_k = T_{tot} \\ \boldsymbol{Ad}-\boldsymbol{h}=\boldsymbol{0} \\ \Phi(\mathbf{q})=\mathbf{0} \\ \boldsymbol{q}_{i,k} \mid \min\{w_i(\boldsymbol{q})\} \end{cases}$$

C. Energy-optimal trajectory planning: results

The methods proposed here are applied to the same planning problem, which is specified as a sequence of 5 via-points located on a straight line in the operative space of the robot. The numerical solution of the optimization problem is, for both methods, obtained using a Sequential Quadratic Programming routine developed in MATLAB. The cost function is evaluated by numerically computing the energy absorbed by each motor, according to Eqs. (8) - (11). A time step equal to 10 ms has been adopted for numerical computation of the integrals. The decision variable vector for both the proposed method and the benchmarks includes the four optimal time intervals $\boldsymbol{T} = [T_1, T_2, T_3, T_4]$ between two consecutive via-points. Additionally, the energy-optimal method includes 5 positions of joint 1 at the 5 via-points.

The result delivered by the proposed energy-optimal method results in the definition of the path shown in Fig. 2. The

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one obtained by the first benchmark method is shown in Fig. 3. Although not shown, the path obtained with the second benchmark is very similar to the latter. The comparison between Figs. 2 and 3 highlights that the distance between the energy-optimal path and the one achieving the best manipulability index is quite small, and both the paths are very close to the ideal straight line. Indeed, the root mean square difference between the ideal straight line and the actual path is 5.18*e*-3 m in both cases.

As far as energy consumption is concerned, the benchmark method results in a total energy requirement equal to 18.10 J and 17.74 J for benchmark #1 and #2, respectively, while the proposed method reduces such a value to just 13.88 J. The energy saving is therefore equal to 23.31 % and to 21.76 %, respectively, despite the fact that the paths appear almost identical in the operative space. Therefore, significant energy savings can be obtained without affecting the spatial path. The analysis of the energy consumption can be inferred by Fig. 4, which shows the electic power $W_i(t) = V_i(t) I_i(t)$ drawn by each motor.

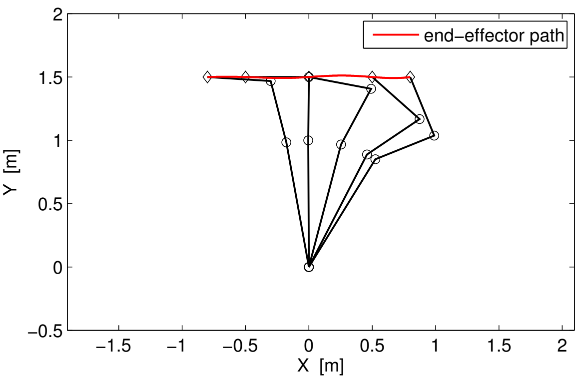

Fig. 2. Optimized trajectory of the proposed method: path and robot poses at via-points

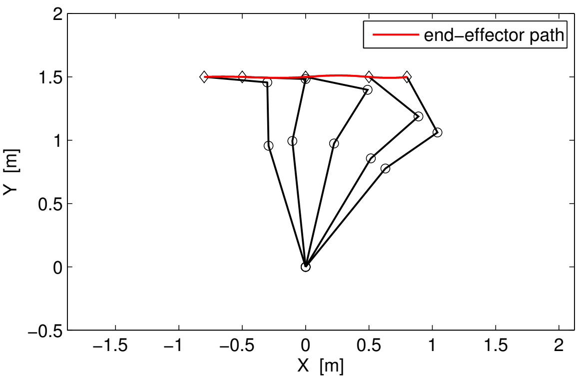

Fig. 3. Optimized trajectory of the benchmark method: path and robot poses at via-points

The red lines refer to the benchmark #1, the blue one to benchmark #2, while the black ones refer to the energy-optimal method. Such plots show that the solution obtained by the energy-optimal approach sensibly reduces the power absorbed by the first motor. Since this motor lies at the base of the robot, it has to drive the largest inertia and hence it has a relevant contribution to the overall energy consumption. A meaningful reduction is also evident for motor 2, in particular at the end of the motion, at the cost of an increase in the power consumption for motor 3. However motor 3 is the one with the smallest energy consumption, due to the smaller torque required to compensate for smaller inertia forces. Motor 3 is also the one in which energy regeneration happens: negative values of the electric power means that it behaves as a generator.

Figure 5 shows the joint speed for the three methods. It clearly reveals that the best solution is achieved by reducing displacement, and hence speed and acceleration, of joint 1. The reduction of such a displacement is compensated by larger displacement of joints 2, and to a lesser extent, and 3.

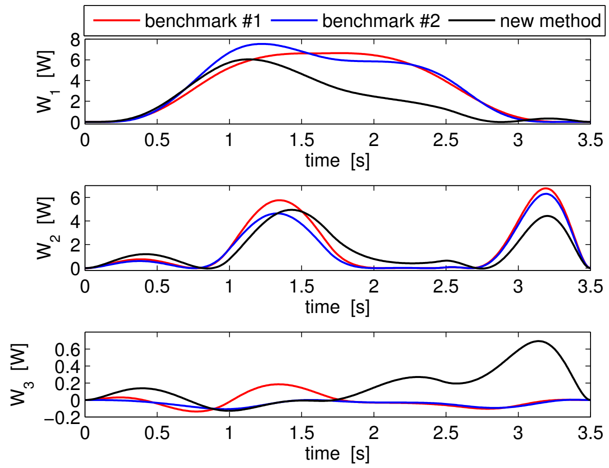

Fig. 4. Optimized trajectories: absorbed electric powers, comparison between the benchmark and the proposed method

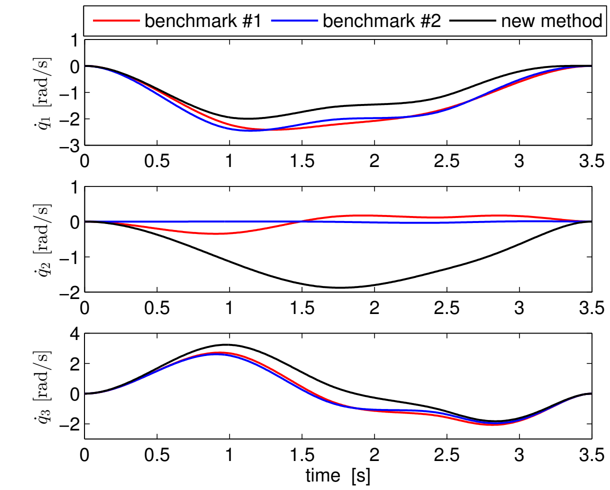

Fig. 5. Optimized trajectories: motor speed, comparison between the benchmark and the proposed method

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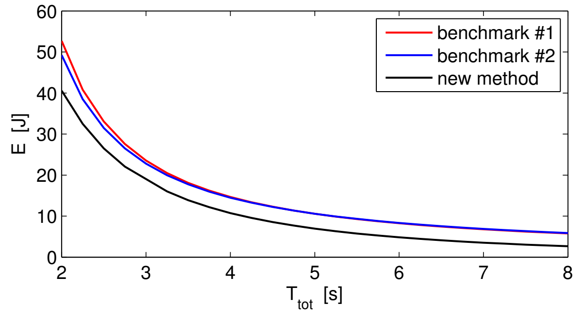

Fig. 6. Total energy consumption vs. total execution time: comparison between the proposed method and two benchmark methods

A further comparison is set by running a sequence of optimization procedures with T_{tot} ranging from 2 to 8 s and by comparing the outcomes of the three mothods. The resulting energy consumptions are then plotted in Fig. 6. Such a picture shows that the proposed method outperforms by a noticeable amount the benchmark ones for any choice of the total execution time, and that the two benchmarks, despite being based on two different performance indexes, provide very similar results.

CONCLUSIONS

This work proposes a novel solution to the problem of computing energy-optimal trajectories for functionally redundant robots. The method exploits the presence of an operational-space dimension that is greater than its operational task-space dimension, to minimize electric energy consumption. The computation of the energy requirement is based on the dynamic model of the robot and of the electric actuators driving the joints; regeneration between actuators is assumed. The trajectory is based on a sequence of spline-based functions ensuring jerk-continuous motion that interpolate a set of desired via-point. Bounded speeds, accelerations and jerks are imposed through constraints, and the overall motion time is imposed. The proposed method selects the solutions of the inverse kinematic problem and the time intervals between two consecutive via-points that minimize energy consumption in the presence of the mentioned constraints. The results presented in the paper show that, for a simple task of a planar 3-dof robot, the proposed method allows for a significant improvement of the energetic efficiency when compared to two meaningful benchmark methods that exploit popular performance index. Given the generality of the models adopted, the proposed method can be successfully applied to arbitrary robot architecture and in the execution of more complicate task exploiting functional redundancy.

REFERENCES

[1] D. Helm, “The european framework for energy and climate policies,” Energy Policy, vol. 64, pp. 29–35, 2014.

[2] D. Meike and L. Ribickis, “Analysis of the energy efficient usage methods of medium and high payload industrial robots in the automobile industry,” in 10th International Symposium Topical Problems in the Field of Electrical and Power Engineering Pärnu, Estonia , 2011.

[3] D. Meike and L. Ribickis, “Energy efficient use of robotics in the automobile industry,” in Advanced Robotics (ICAR), 2011 15th International Conference on . IEEE, 2011, pp. 507–511.

[4] M. Hazan, L. Embon, and R. Madvil, “Method and apparatus for saving energy and reducing cycle time by using optimal robotic joint configurations,” Mar. 29 2016, US Patent 9,298,863.

[5] M. Frische and F. Jungnickel, “Reducing energy consumption of industrial robots by using new methods for motion path programming,” May 16 2017, US Patent 9,649,765.

[6] G. Carabin, E. Wehrle, and R. Vidoni, “A review on energy-saving optimization methods for robotic and automatic systems,” Robotics , vol. 6, no. 4, p. 39, 2017.

[7] E.-C. Corporation, DC Motors, Speed controls, servo systems: An engineering handbook . Pergamon Press.

[8] P. M. Ho, N. Uchiyama, S. Sano, K. Sawada, A. Kato, and T. Yonezawa, “Simple motion trajectory generation for energy saving of industrial machines,” in System Integration (SII), 2012 IEEE/SICE International Symposium on . IEEE, 2012, pp. 476–480.

[9] P. Boscariol, G. Carabin, A. Gasparetto, N. Lever, and R. Vidoni, “Energy efficient point-to-point trajectory generation for industrial robotic

[10] D. Richiedei and A. Trevisani, “Analytical computation of the energy-efficient optimal planning in rest-to-rest motion of constant inertia systems,” Mechatronics , vol. 39, pp. 147–159, 2016.

[11] K. Inoue, K. Ogata, and T. Kato, “An efficient power regeneration and drive method of an induction motor by means of an optimal torque derived by variational method,” IEEJ Transactions on Industry Applications , vol. 128, pp. 1098–1105, 2008.

[12] K. Inoue, Y. Asano, K. Kotera, and T. Kato, “Optimal energy saving trajectories of induction motor with suppression of sudden acceleration and deceleration,” in Energy Conversion Congress and Exposition (ECCE), 2014 IEEE, pp. 3219–3223.

[13] J. T. Betts, “Survey of numerical methods for trajectory optimization,” Journal of Guidance control and dynamics , vol. 21, no. 2, pp. 193–207, 1998.

[14] Y. Halevi, E. Carpanzano, G. Montalbano, “Minimum energy control of redundant linear manipulators”, Journal of Dynamic Systems, Measurement, and Control 2014, vol. 136, pp. 051016- - 051016-8.

[15] J. Angeles, ”Fundamentals of robotic mechanical systems (Vol. 2)”, Springer-Verlag 2002.

[16] D. Corinaldi, J. Angeles, and M. Callegari. "Posture optimization of a functionally redundant parallel robot." Advances in Robot Kinematics 2016. Springer, pp. 101-108, 2018

[17] L. Biagiotti and C. Melchiorri, Trajectory planning for automatic machines and robots. Springer Science & Business Media, 2008.

[18] K. Petrinec and Z. Kovacic, “Trajectory planning algorithm based on the continuity of jerk,” in Control & Automation, 2007. MED’07. Mediterranean Conference on . IEEE, 2007, pp. 1–5.

[19] C. Hansen, K. Eggers, J. Kotlarski, and T. Ortmaier, “Comparative evaluation of energy storage application in multi-axis servo systems,” in Proceedings of the 14th IFToMM World Congress, Taipei, Taiwan , 2015.

[20] J. Park, “Motion profile planning of repetitive point-to-point control for maximum energy conversion efficiency under acceleration conditions,” Mechatronics, vol. 6, no. 6, pp. 649–663, 1996.

[21] Z. Yu, C. Han, and M. Haihua, “A novel approach of tuning trapezoidal velocity profile for energy saving in servomotor systems,” in Control Conference (CCC), 2015 34th Chinese . IEEE, 2015, pp. 4412–4417.

[22] B. Siciliano, L. Sciavicco, L. Villani, G. Oriolo, Modelling and control of robot manipulators. Springer Science & Business Media, 2012

P. Boscariol, D. Richiedei. *Spline-based energy-optimal trajectory planning for functionally redundant robots*, 2018 14th IEEE/ASME International Conference on Mechatronic and Embedded Systems and Applications (MESA),
DOI: 10.1109/MESA.2018.8449155