A variational approach for the reduction of transient load sway in overhead cranes

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Abstract

The operation of overhead cranes requires to reduce the load oscillations that arise both during the motion and after it ends. While the issue of ensuring zero residual vibrations has been widely investigated, less attention has been paid to transient oscillations. This work investigates the possibility of solving both problems using the tool of model-based motion planning, and in particular by means of a variational approach. The variational formulation allows for the synthesis of a crane motion that results in negligible residual oscillations and reduced peak values of the crane sway angle during motion. A comparison with the input shaping technique shows the practical advantages offered by the proposed method.

1 Introduction

The operation of overhead cranes requires to deal with the problem of damping or eliminating the load oscillations that naturally occur during and after the motion. Such a problem is usually tackled both in literature and in industrial practice through closed-loop control or through open-loop control. The latter approach consists in designing optimal command profiles that move the load to the desired position without residual load oscillations. Focusing on trajectory planning can, in this sense, reduce the need for an elaborate control, which can be an useful feature in industrial application when the use of accurate sensors can be impractical or non cost-effective.

The problem of motion-induced vibration is quite evident in machines for which the common rigid link assumption is not valid [1], as well as in machines built with flexible elements, such as ropes or springs, or underactuated systems. Such systems might incur in high vibration not only during the motion, but also after motion completion [2, 3]. This problem is particularly relevant for underactuated multibody systems, i.e. those having less control inputs than degrees of freedom. This is typical in the aforementioned cases of systems with joint or link flexibility. In such cases the control problem is also exacerbated because of their underdamped behavior that imposes either open-loop or closed-loop if the motion specifications are defined for unactuated degrees of freedom.

The trajectory planning problem for underdamped and underactuated systems has been analyzed and solved using a large number of techniques.

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An example is [4], which describes the analytical expression of a trajectory, parametrized as a s-curve speed profile, or [5], which employs smoothed jerk profiles. Inverse dynamics approaches are also often adopted to compute the optimal profile of the forces of actuated degrees of freedom.

Concepts similar to the ones used in [4] and [5] are exploited also for the class of methods referred to as input shaping, which have gained a wide diffusion due to their simple implementation [6, 7]. Input shaping filters can be used to produce rest-to-rest motion with zero residual vibration for single mode [8] and multi-mode systems as well [9]. Such techniques have also been extended to react to the change of the system frequency over time, leading to the definition of robust shapers and to extra-insensitive robust shapers [10]. Such techniques represent a de-facto standard in the industrial application control through open-loop approaches.

The technique used in this work is, in contrast, based on the calculus of variation. The trajectory planning problem is set up as an optimization problem based on the dynamics of the system. Using variational calculus, the necessary conditions for optimality are imposed using the Pontryagin's Minimum Principle (PMP). That transforms the optimization problem into a Two-Point Boundary Value Problem (TP-BVP), whose solution is accomplished numerically. The use of a variational formulation and a specific choice of the underlying cost function allows to shape the motion law by taking into account not only the amplitude of residual vibrations, but also the amplitude of the load oscillation during the transient. The latter is a feature that is not shared by shaping filters, so that the currently proposed method is specifically aimed at solving this shortcoming. Numerical results are presented to show the application of the proposed method to a benchmark problem.

2 Variational formulation to motion planning

The aim of this work is to propose and test a model-based solution to the design of motion profiles for overhead cranes that result in null residual vibration and in a reduced sway angle during motion. A variational approach is adopted by starting from the dynamic model of the system represented through differential equations. The dynamics of a crane can be represented as:

$$\ddot{\theta}(t) = -\frac{g}{L}\sin\theta(t) - \frac{\ddot{y}(t)}{L} \tag{1}$$

in which $\theta(t)$ is the load sway angle, $\ddot{y}(t)$ is the linear acceleration of the cart, the constant rope length is L and g is the gravity acceleration constant. The simple model of Eq. (1) can be augmented to account for the actuator bandwidth, which is described by the time constant τ, by defining a fictitious input $u(t)$, that is the acceleration reference signal, so that the cart acceleration is defined by:

$$\ddot{y}(t) = u(t) - \tau\dddot{y}(t) \tag{2}$$

Since a first-order description is needed for solving the variational problem in lieau of the second-order one of Eq. (1), the following state vector is used:

$$\mathbf{x}(t) = \left[\ddot{y}, \dot{\theta}, \dot{y}, \theta, y\right]^T \tag{3}$$

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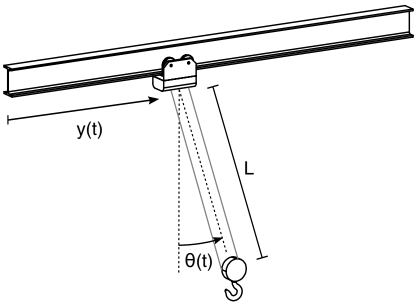

Figure 1: Kinematic model of an overhead crane

The resulting nonlinear dynamic model therefore is:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -\frac{\ddot{y}(t)}{\tau} + \frac{u}{L} \\ -\frac{\ddot{y}(t)}{\tau} - \frac{g}{L}\sin\theta(t) \\ \ddot{y}(t) \\ \dot{\theta}(t) \\ \dot{y}(t) \end{bmatrix} \tag{4}$$

The formulation of the dynamic model of Eq. (4) is needed to define the crane motion planning algorithm as a Two-Point Boundary Value Problem, which consist in the minimization of a cost functional with bounds on the values of the system state at initial and final instants of the motion, i.e. at the two boundaries. In a general formulation, a TPBVP is defined upon a set of first-order differential equations that are defined as:

$$\dot{\mathbf{x}}(t) = \mathbf{\Omega}(\mathbf{x}, u, t) \tag{5}$$

The the motion law is shaped by the choice of a scalar cost function that is to be minimized, defined as:

$$J = \int_{t_0}^{t_f} f(\mathbf{x}, u, t)dt \tag{6}$$

with desired initial and final states $\mathbf{x}(t_0)$ and $\mathbf{x}(t_f)$ as the left and right, respectively, boundary conditions. The boundary conditions can be used, in a trajectory planning framework, to impose the desired initial and final kinematic conditions, such as initial and final positions and velocities. Under this premises, a trajectory planning problem can be translated into a minimization problem, aimed at the minimization of the cost functional J, constrained to the left and right boundary conditions and to the dynamic model of Eq. (5). The solution to this class of problem can be achieved using the well-known Pontryagin Minimum Principle (PMP), following a procedure that will be briefly recalled here. First of all, the Hamiltonian of the system can be defined as:

$$\mathcal{H}(\mathbf{x}, \boldsymbol{\lambda}, u, t) = f + \boldsymbol{\lambda}^T(t)\mathbf{\Omega}(\mathbf{x}, u, t) \tag{7}$$

The Hamiltonian is a scalar function, that requires the definition of a set of auxiliary functions $\lambda_i(t)$, often called co-states or Lagrangian functions.

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The Lagrangians are as many as the number of independent equations in $\boldsymbol{\Omega}(\mathbf{x}, u, t)$. The necessary conditions for finding a minimum of the minimization problem stated above are, according to the PMP:

$$\frac{\partial \mathcal{H}}{\partial u} = 0 \tag{8}$$

$$\dot{\mathbf{x}} = \frac{\partial \mathcal{H}}{\partial \lambda} \tag{9}$$

$$\dot{\boldsymbol{\lambda}} = -\frac{\partial \mathcal{H}}{\partial \mathbf{x}} \tag{10}$$

The solution of eq. (8), that is often referred as the 'optimal control' $u^*(t)$, can be used to define the minimizing Hamiltonian $\mathcal{H}^*(\mathbf{x}, \boldsymbol{\lambda}, u^*, t)$. By using $\mathcal{H}^*$, an augmented system of ODEs can be defined as:

$$\dot{\mathbf{z}} = \begin{bmatrix} \dfrac{\partial \mathcal{H}^*}{\partial \boldsymbol{\lambda}} \\ -\dfrac{\partial \mathcal{H}^*}{\partial \mathbf{x}} \end{bmatrix} \tag{11}$$

The equation (11) essentially states that the dynamics of the augmented state vector:

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \boldsymbol{\lambda}(t) \end{bmatrix} \tag{12}$$

must comply simultaneously with the three PMP conditions, and that each solution of this ODE system that complies with the prescribed boundary condition, automatically minimizes the cost functional J as defined in eq. (6). This method therefore can be applied to any dynamic model, provided that the system dynamics is sufficiently differentiable to define the conditions shown so far. Nonlinear models can be used as well, so that the application of the of the linearization procedures that are used in most shaping techniques is not required.

In literature and in engineering practice, two-point boundary value problems are solved numerically, using general-purpose computing environments such as MATLAB [11] or specific software tools such as `psopt` [12], by using either collocation, a shooting algorithm or a pseudo-spectral method.

3 Maximum load sway reduction

The variational formulation can be effectively used to shape the dynamic behavior of the crane when performing a rest-to-rest motion. Unlike most works available in literature, which focus on the elimination or the reduction of the load sway after motion completion [7], here the target is to achieve also a reduction of the transient load sway, i.e. to limit the oscillation of the suspended load during motion. This feature can be very useful to avoid excessive load sway angles that might cause collisions. The rest-to-rest conditions and the prescribed translation define the 'left' and 'right' boundary conditions $\mathbf{x}(t_0)$ and $\mathbf{x}(t_f)$, while the cost function f that appears in Equations (6-7) can be used to shape the actual motion profile. The cost function chosen in this work is designed as:

$$f = \frac{1}{2}u^2(t) + \beta \exp\left((\gamma\theta(t))^2\right) \tag{13}$$

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Table 1: Performance measurements

Test	$\ddot{y}_{max}$ $[m/s^2]$	$\ddot{y}_{RMS}$ $[m/s^2]$	θ_{max} [rad]	θ_{RMS} [rad]	$\theta_{residual}$ [rad]
poly5 + ZV shaper	0.6441	0.4006	0.0868	0.0533	$2.5275{\cdot}10^{-4}$
cheap control	0.6448	0.3236	0.0752	0.0479	$3.6546{\cdot}10^{-7}$
variational, $\beta = 10$	0.8379	0.3354	0.0657	0.0429	$6.7558{\cdot}10^{-7}$
variational, $\beta = 20$	0.9797	0.3757	0.0630	0.0408	$8.6966{\cdot}10^{-7}$
variational, $\beta = 80$	1.3978	0.5014	0.0601	0.0374	$1.4908{\cdot}10^{-6}$

The choice of the formulation in Eq. (13) is motivated by the need to introduce a strong penalty of peak values of the load sway during motion, and for this reason an exponential weighting function is included. The coefficient γ is included to ensure a proper scaling of the sway angle θ (which is measured in radians), while the weight β is used to specify the balance between the two components of the cost function. The first component is commonly referred to as the 'cheap control', and is used to mitigate the actuator effort, while the second one is chosen to reduce the peak values of the sway angle. The tradeoff between the two effects is set by choosing a proper value for the weighting parameter β. In the numerical results the scaling factor γ has been set equal to 4, i.e. a value that has shown to provide an easy tuning of the cost function by means of the single parameter β.

4 Numerical results

The proposed variational approach and the choice of the cost function of Eq. (13) is here tested to show the effectiveness of the method for the planning of rest-to-rest motion profiles with limited load swing. In order to do so, a benchmark crane has been designed by setting the rope length L equal to 1 m, so that the sway oscillation frequency is 0.498 Hz. The planning problem is set so that the motion of the overhead crane performs a rest-to-rest translation equal to 0.5 m in 2.5 s.

To provide a comparison with widely adopted methods, a comparison is set by showing the effects of varying values of the weight β together with the results that can be obtained with the 'cheap control' approach, as well as with the application of a Zero Vibration (ZV) shaping filter. The responses of the crane to the open-loop application of the planned motion profiles are evaluated by some performance indexes whose values are reported in Table 1. The chosen performance indexes include the peak and the root-mean-square values of the cart acceleration $\ddot{y}(t)$ and of the load swing angle $\theta(t)$ as recorded during the whole motion duration. Peak values of residual vibration are reported as well.

The analysis of the data in Table 1 shows that the least actuator effort is achieved, as expected, by the cheap control approach, that is obtained within the proposed variational approach simply by setting to zero the weight β. Increasing values of β result in a more pronounced actuator effort, as testified by the higher values of both peak values and RMS values of the cart acceleration $\ddot{y}$ during the transient. This feature is compensated by the lower values of both peak and RMS load swing angles that gets more evident with increasing values of β. The tradeoff between

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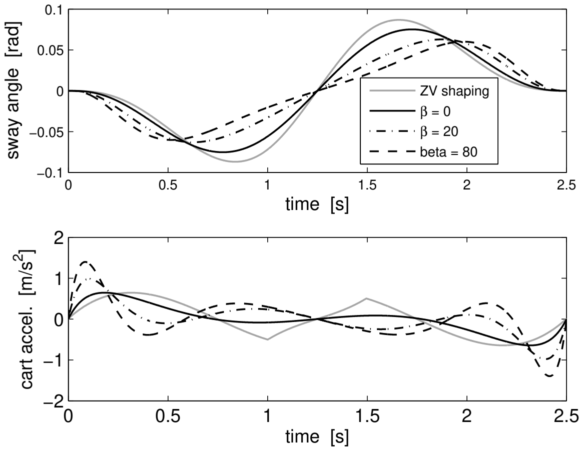

Figure 2: Comparison between ZV shaper and variational solution with various β values: sway angle and cart acceleration

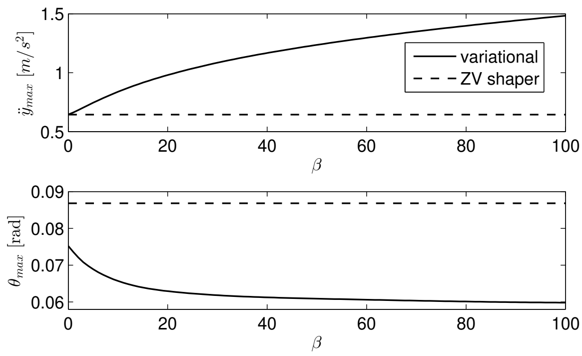

Figure 3: Peak value of cart acceleration and load sway angle vs. weight β

the increase of the actuator effort and the reduction of the transient load oscillation is more precisely measured by looking at Fig. 3, which shows the direct and the inverse proportionality between the value of β, $\ddot{y}_{max}$ and θ_{max}, respectively.

The tradeoff between the two is measured also in terms of ratios between their RMS and peak values, according to the data displayed in Fig. 4. Such a plot can be used to highlight the effects of β on the shaping of the main motion profiles features, as well as to provide a qualitative guideline when choosing a suitable value for β.

5 Conclusion

In this paper a method for the reduction of the transient load sway and the elimination of residual oscillations in overhead cranes has been presented. The method applied to underactuated systems with pronounced oscillation phenomena. The application of the proposed solution to a benchmark

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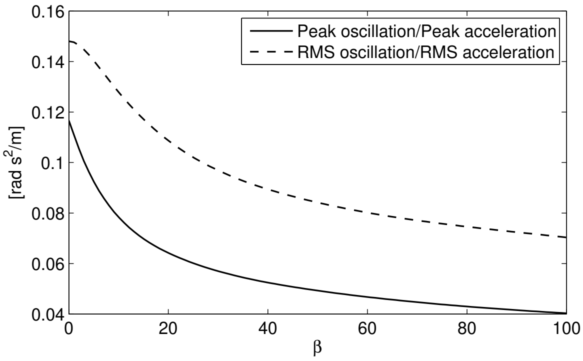

Figure 4: Ratios of peak and RMS values of load oscillation and cart acceleration vs. weight β

problem has highlighted the effective capability in reducing both the peak and the RMS values of the load sway angle. The comparison with other commonly used method, such as a 'cheap control' formulation or a ZV input shaping filter shows that the custom defined cost function and the variational formulation can be valuable tools when an optimal tradeoff between the actuator effort and the maximum amplitude of the load sway is required.

References

[1] Biagiotti L, Melchiorri C (2008) Trajectory planning for automatic machines and robots. Springer Science & Business Media

[2] Abe A (2016) An effective trajectory planning method for simultaneously suppressing residual vibration and energy consumption of flexible structures. Case Studies in Mechanical Systems and Signal Processing 4:19–27

[3] Carabin G, Wehrle E, Vidoni R (2017). A Review on Energy-Saving Optimization Methods for Robotic and Automatic Systems. Robotics, 6(4), 39

[4] Uchiyama N, Sano S, Haneda K (2015) Residual vibration suppression using simple motion trajectory for mechanical systems. SICE Journal of Control, Measurement, and System Integration 8(3): 195–200

[5] Bearee R (2014) New Damped-Jerk trajectory for vibration reduction. Control Engineering Practice 28:112–120

[6] Kiang CT, Spowage A, Yoong CK (2015) Review of control and sensor system of flexible manipulator. Journal of Intelligent & Robotic Systems 77(1):187–213

[7] Singhose W (2009) Command shaping for flexible systems: A review of the first 50 years. International journal of precision engineering and manufacturing 10(4):153–168.

This is a pre-print of the article: Boscariol P., Richiedei D. (2019) A Variational Approach for the Reduction of Transient Load Sway in Overhead Cranes. In: Carbone G., Gasparetto A. (eds) Advances in Italian Mechanism Science. IFToMM ITALY 2018. Mechanisms and Machine Science, vol 68. Springer, Cham https://doi.org/10.1007/978-3-030-03320-0_49

[8] Singer NC, Seering WP (1990) Preshaping command inputs to reduce system vibration. Journal of Dynamic Systems, Measurement, and Control 112(1):76–82

[9] Díaz IM, Pereira E, Feliu V, Cela JJ (2010) Concurrent design of multimode input shapers and link dynamics for flexible manipulators. IEEE/ASME Transactions on Mechatronics 15(4):646–651.

[10] Pao LY, Chang TN, Hou E (1997) Input shaper designs for minimizing the expected level of residual vibration in flexible structures. In: Proceedings of the 1997 American Control Conference, IEEE

[11] Shampine LF, Gladwell I, Thompson S (2003) Solving ODEs with MATLAB. Cambridge University Press

[12] Becerra VM (2010) Solving complex optimal control problems at no cost with PSOPT. In Computer-Aided Control System Design (CACSD), 2010 IEEE International Symposium on, pp. 1391.-1396

This is a pre-print of the article: Boscariol P., Richiedei D. (2019) A Variational Approach for the Reduction of Transient Load Sway in Overhead Cranes. In: Carbone G., Gasparetto A. (eds) Advances in Italian Mechanism Science. IFToMM ITALY 2018. Mechanisms and Machine Science, vol 68. Springer, Cham https://doi.org/10.1007/978-3-030-03320-0_49