Robust model-based trajectory planning for flexible mechanisms: experimental assessment

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Abstract

This paper presents the experimental validation of a robust model-base motion planning technique for underactuated flexible mechanical systems in point-to-point motion. The method is aimed at producing motion profiles that result in reduced transient and residual vibrations, so that the motion task is completed with an accurate positioning. Unlike other methods in literature, the proposed method is also targeted at robustness, i.e. it reduces the effect of the uncertainties of the model used in the trajectory design. The preliminary experimental results proposed here show the reduced vibrations produced by the robust motion profile when the flexible-link mechanism is perturbed by increasing the endpoint mass, and its advantages over traditional input shaping techniques.

1 Introduction

High-speed operation of mechanisms and robots often result in pronounced vibration that might occur not only during motion, but also after its completion, because of flexibility of joints and link. Vibration phenomena, if neglected or poorly cared of, can worse the positioning accuracy adn can also lead to premature wear or mechanical failures. This topic has been widely investigated by scientist and practitioners in the last decades. According to the literature, the reduction of motion-induced vibrations can be deal with either by focusing on closed-loop control, which can damp vibration, either through the design or the alteration of motion profiles that minimally excite vibration phenomena.

Countless examples of closed-loop methods have been proposed for the vibration reduction in mechanical systems, according to the literature reviews on the topics of vibration control of flexible mechanisms [1]. Closed-loop controls can be very effective, but their design can be challenging and their implementation might require high computational requirements and dedicated sensors.

If vibration reduction is dealt with by motion design, one of the most commonly used methods is input-shaping [2], which has proved to be of simple implementation and high effectiveness in many practical situations. One or more shaper [3] can be used to filter a generic motion profile to obtain a motion with null residual vibration under ideal identification of the modal frequencies of the system. Over the last decades several input shaping methods have been proposed, notably the Zero-Vibration (ZV) shaper and the Zero Derivative (ZVD) shaper [4] have proved to be effective in many situations.

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Alternatively, a motion profile can be designed to provide minimal excitation of motion-induced vibrations, either by enhancing smoothness [5, 6] or through a proper model-based design [7]. Another method can be found in [8], which describes the analytical expression of a trajectory, parametrized as a s-curve speed profile, and suggests a tuning for minimizing residual vibrations, or [9], which proposes to achieve similar results by the use of a smoothed jerk profiles.

An approach based on variational calculus for the design of rest-to-rest motion of flexible mechanism is proposed and validated in this work, by exploiting the general theory developed in [12] and extended in [13]. This work is aimed at testing the application of this method to the design of robust rest-to-rest motion profiles for flexible-link mechanism with reduced sensitivity to model-plant mismatches, so that the differences between the theoretical and the real behavior of the system result in a mild deterioration of performances. This capability is incorporated within a framework based on a variational formulation, being the motion design problem cast as an optimization problem constrained to the dynamics of the plant for which the motion is planned. Experimental results will provide an evidence of the effectiveness of the proposed method in reducing the amplitude of residual vibration when high-speed motion of flexible systems is performed.

2 Variational formulation to motion planning

2.1 System model

The variational method already presented in [12] and [13] will be here recalled, together with the description of the model used to describe the dynamics of the testbed. The testbed comprises a clamped aluminum beam that is mounted on the end-effector of an Adept Quattro s650h robot, as shown in Fig. 1. The latter is operated along a straight line, so that it emulates the behavior of a cart moving along a linear guide, and the motion of the beam is constrained to a single plane. The dynamic model of the flexible mechanism can be effectively represented by a FEM model, and hence through its mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$, which can be defined according to the Euler-Bernoulli beam formulas. The system can be represented by means of the position along the Y direction of the end-effector of the robot, $y(t)$, and the nodal displacements of the beam, with reference to a moving reference frame. Such a moving reference is defined as an Equivalent Rigid Link System (ERLS), whose motion is forced to track the real mechanism by setting no relative translation at the clamping end of the beam. The elastic relative displacements, including both nodal translations and rotations of the N finite elements are collected in vector $\mathbf{u}(t)$:

$$\mathbf{u} = [u_1, u_2, \dots, u_{2N}]^T \tag{1}$$

The overall motion of the system can be represented by $2N + 1$ differential equations as:

$$\begin{bmatrix} \mathbf{M} & \mathbf{MS} \\ \mathbf{S}^T\mathbf{M} & \mathbf{S}^T\mathbf{MS} + M_c \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ y \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} F_y \tag{2}$$

where F_y is the force exerted by the the robot on end effector, whose mass is M_c, along the Y direction and $\mathbf{S}$ is the matrix of the nodal sensitivity coefficients.

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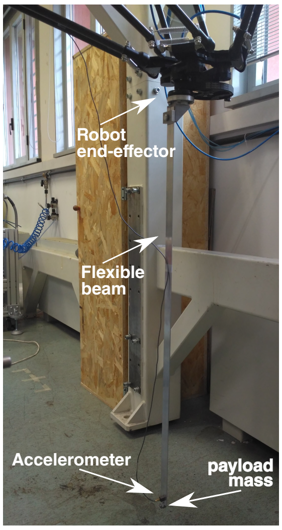

Figure 1: The experimental setup: the flexible beam mounted on the Adept Quattro robot

Table 1: Mechanical parameters of the flexible beam

Parameter	Symbol	Value	
Young's modulus	E	$68 \cdot 10^9$	$[kg\, m^{-1} s^{-2}]$
Beam width	b	$30 \cdot 10^{-3}$	$[m]$
Beam thickness	h	$2 \cdot 10^{-3}$	$[m]$
Beam length	L	0.925	$[m]$
Mass density	ρ	$2.583 \cdot 10^3$	$[kg\, m^{-3}]$
Tip mass	m_p	0.01	$[kg]$

In this case $\mathbf{S}$ is constant owing to the simple kinematics of the mechanism under consideration, but in general it is a function of the ERLS general coordinates and hence it changes the modal properties of the system [14]. Whenever $M_c \gg \mathbf{S}^T\mathbf{MS}$ the robot dynamics is unaffected by the beam elastic dynamics. Accordingly, the dynamic model of Eq. 2 can be rewritten by considering $y(t)$ as an exogenous input of the system made by the second order differential equations that govern the flexible behavior of the beam:

$$\mathbf{M\ddot{u}}(t) + \mathbf{Ku}(t) = -\mathbf{MS}\ddot{y}(t) \tag{3}$$

Additionally, the model in Eq. 3 can be improved to represent the robot position control bandwidth through the time constant τ. The input of this augmented model is the cart acceleration reference signal $\nu(t)$, so that the actual cart acceleration is defined by:

$$\ddot{y}(t) = \nu(t) - \tau \dddot{y}(t) \tag{4}$$

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The need for a first-order ODE description of the system, as required by the variational formulation suggests to re-define the state vector as:

$$\mathbf{x}(t) = \left[\ddot{y}(t), \dot{\mathbf{u}}(t), \dot{y}(t), \mathbf{u}(t), y(t)\right]^T \tag{5}$$

and therefore the complete dynamic model of the plant can be stated as:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -\frac{1}{\tau} & \mathbf{0} & 0 & \mathbf{0} & 0 \\ \mathbf{S} & \mathbf{0} & \mathbf{0} & -\mathbf{M}^{-1}\mathbf{K} & \mathbf{0} \\ 1 & \mathbf{0} & 0 & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & 1 & \mathbf{0} & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} -\frac{1}{\tau} \\ \mathbf{0} \\ 0 \\ \mathbf{0} \\ 0 \end{bmatrix} \nu(t) := \mathbf{\Omega}(\mathbf{x}, \nu, t) \tag{6}$$

2.2 Motion design under nominal conditions

The dynamic model of Eq. 6 can be used, in the proposed framework, to set-up and solve the motion design problem as a so-called Mayer problem, i.e. the minimization of the cost function:

$$J = \int_{t_0}^{t_f} f(\mathbf{x}, \nu, t) dt \tag{7}$$

in the presence of the differential constraint:

$$\dot{\mathbf{x}}(t) = \mathbf{\Omega}(\mathbf{x}, \nu, t) \tag{8}$$

and of boundary conditions:

$$\begin{aligned} \mathbf{x}(t_0) &= \mathbf{x}_0 \\ \mathbf{x}(t_f) &= \mathbf{x}_f \end{aligned} \tag{9}$$

Appropriate boundary conditions can be used, in this case, to ensure rest-to-rest motion within a prescribed overall displacement in the time interval $t \in [t_0, t_f]$. The mentioned Mayer problem can be solved, according to the Pontryagin Minimum Principle (PMP) [15] starting from the definition of the Hamiltonian function as:

$$H(\mathbf{x}, \boldsymbol{\lambda}, \nu, t) = f + \boldsymbol{\lambda}^T \mathbf{\Omega}(\mathbf{x}, \nu, t) \tag{10}$$

The scalar Hamiltonian function is defined upon a set of auxiliary functions $\lambda_i(t)$, commonly referred as Lagrangian functions or co-states. According to the PMP the necessary conditions for the solution of the Mayer problem defined by Equations 7-9 are:

$$\frac{\partial H}{\partial \nu} = 0; \qquad \dot{\mathbf{x}} = \frac{\partial H}{\partial \boldsymbol{\lambda}}; \qquad \dot{\boldsymbol{\lambda}} = -\frac{\partial H}{\partial \mathbf{x}}; \tag{11}$$

The solution of the first of Eq. 11 is the optimal profile $\nu^*(t)$, which can be used in Eq. 10 to define the minimizing Hamiltonian $H^*(\mathbf{x}, \boldsymbol{\lambda}, \nu^*, t)$. The latter is used to define the augmented system of ODEs:

$$\dot{\mathbf{z}} = \begin{bmatrix} \dfrac{\partial H^*}{\partial \boldsymbol{\lambda}} \\ -\dfrac{\partial H^*}{\partial \mathbf{x}} \end{bmatrix} \tag{12}$$

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Equation 12 essentialy states that the augmented vector $\mathbf{z} = \left[\mathbf{x}^T, \mathbf{y}^T\right]^T$ must comply simultaneously with the PMP conditions. This method can be applied to any dynamic system that can be represented by a set of first-order ordinary differential equations. To ensure good numerical conditioning of the model, its dimension should be kept reasonably small, either by using a small number of finite elements, or through a careful model reduction [10, 11].

2.3 Motion design under robust conditions

The trajectory designed in Section 2.2 ensures fast motion and reduced residual vibration, after the motion completion, provided that the dynamic model used for the planning stage is sufficiently accurate. However, it is expected that the amplitude of residual vibration will be larger in the presence of a sensible mismatch between the model used for the planning and the actual plant. If the mismatch is due to the alteration of a single parameter of the plant, accounting for this occurrence is equivalent to enhancing the parametric robustness of the motion design.

As already proposed and explained in further detail in [12], the robustness to parametric perturbations can be boosted by augmenting the optimization problem expressed by Equations 7-9 with the sensitivity functions of the dynamic systems, i.e. the partial derivatives of state vector $\mathbf{x}(t)$ made with reference to the uncertain parameters. Supposing that the uncertain or perturbed parameter is the tip mass m_p, an augmented state vector can be defined as:

$$\mathbf{x}^*(t) = \begin{bmatrix} \mathbf{x}(t) \\ \dfrac{\partial \mathbf{x}(t)}{\partial m_p} \end{bmatrix} \tag{13}$$

The dynamics associated with this augmented state vector will be, therefore:

$$\dot{\mathbf{x}}^*(t) = \begin{bmatrix} -\frac{1}{\tau} & \mathbf{0} & 0 & \mathbf{0} & 0 & \mathbf{0} & \mathbf{0} \\ \mathbf{S} & \mathbf{0} & \mathbf{0} & -\mathbf{M}^{-1}\mathbf{K} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 1 & \mathbf{0} & 0 & \mathbf{0} & 0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & 1 & \mathbf{0} & 0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\frac{\partial \mathbf{M}^{-1}\mathbf{K}}{\partial m_p} & \mathbf{0} & -\mathbf{M}^{-1}\mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \dot{\mathbf{u}} \\ \dot{y} \\ \mathbf{u} \\ y \\ \frac{\partial \dot{\mathbf{u}}}{\partial m_p} \\ \frac{\partial \mathbf{u}}{\partial m_p} \end{bmatrix} + \begin{bmatrix} -\frac{1}{\tau} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \nu \tag{14}$$

Equation 2.3 can be evaluated starting form the analytical description of the 'nominal' ODE system of Eq. 6. The procedure already used to set-up and solve the Mayer problem can now be applied to the 'robustified' system, taking care of including suitable additional boundary conditions. The robustness of the planned trajectory can be enhanced by forcing the value of the sensitivity function to be zero at both initial and final time, as a way to ensure that the fulfillment of zero residual vibration conditions is minimimally affected by model-plant mismatches.

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3 Motion planning for the flexible-link mechanism: nominal and robust design

The dynamic model of Eq. 6 can be used to set-up an solve the TP-BVP associated with the desired rest-to-rest motion design. The design specification include the following specifications: total displacement $h = 0.4$ m, overall motion time $t_f = 1$ s, null vibration at both initial time t_0 and final time t_f, as well as null cart acceleration at the same time instants. Such specifications are translated into the following boundary conditions to the system state:

$$\mathbf{x}(t_0) = [0,0,0,0,0,0,0]^T\,; \quad \mathbf{x}(t_f) = [0,0,0,0,0,0,0.4]^T \tag{15}$$

The cost functional, according to the notation of Eq. 7, is chosen to be $f = \frac{1}{2}\nu^2$, so that the optimization problem will minimize the integral of the square norm of the acceleration reference signal $\nu(t)$ evaluated for the whole duration of the trajectory. The solution of the TP-BVP problem can be obtained numerically, using Matlab's `bvp5c` solver. The resulting motion profile is show in terms of cart position and speed in Fig. 2.

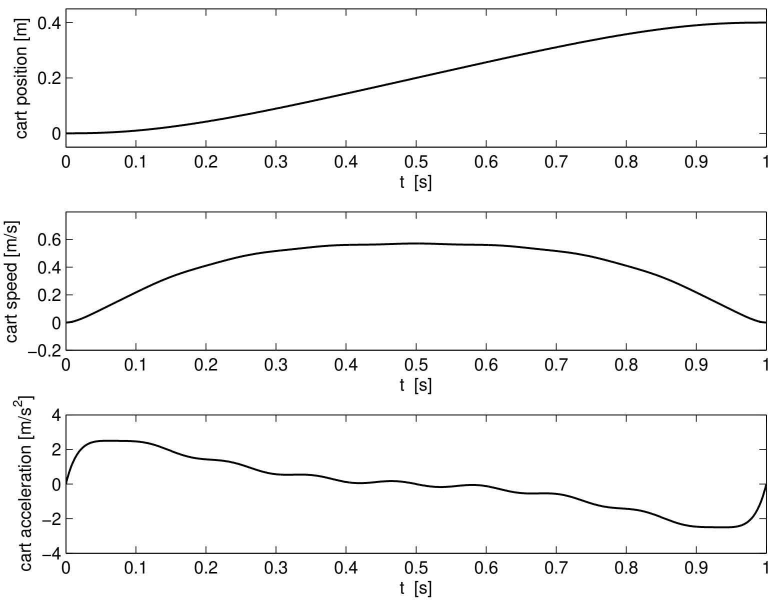

Figure 2: Variational approach: planned motion profile

The 'robustified' counterpart of the planned cart motion profile already presented in Fig. 2 is shown in Fig. 3: the analysis of the speed profile shows that the inclusion of additional constraints has the disadvantage of requiring higher peak speed and acceleration, but in both cases it is expected that reproducing such motion profile will lead to null vibrations when ideal and real plant are equivalent.

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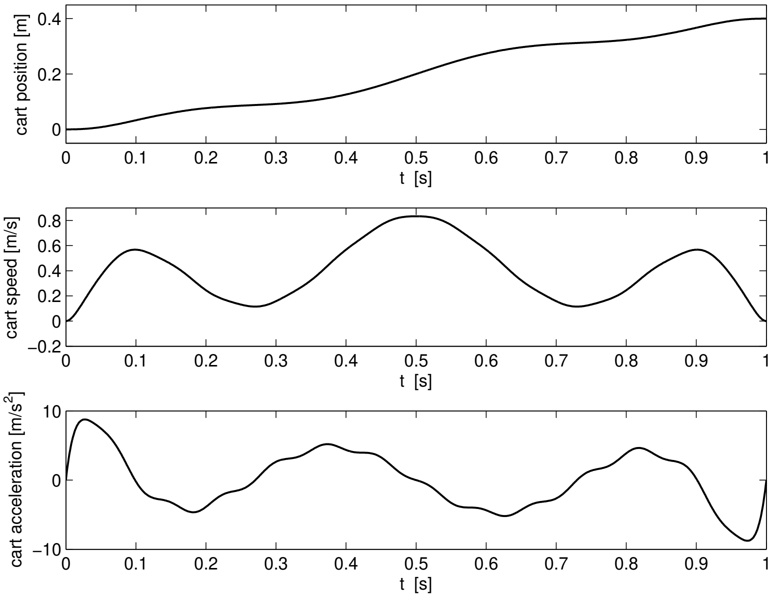

Figure 3: Robust variational approach: planned motion profile

4 Experimental results

The results presented in this section are meant to provide an experimental validation of the proposed robust trajectory planning method, which is performed by feeding the robot controller with the motion profiles presented in Fig. 2 and 3. In a first set of tests a nominal plant is used; in the second one the plant is perturbed by increasing the payload mass mounted on the tip of the beam from 10 g to 25 g.

As a benchmark, the trajectory produced by a fifth-order polynomial trajectory shaped by a Zero-Vibration (ZV) shaper and a Zero Derivative (ZVD) shaper [16] have been tested as well. The shaper are tuned to suppress the first two modes of the beam, which are located at 1.37 Hz and 8.78 Hz. Higher order modes are neglected, since they result in tip oscillation with limited amplitude and fast decay. The ZV shaper method aims at producing a trajectory with zero residual vibration under nominal conditions, while the ZVD shaper is designed to be robust to the variation of natural frequencies. The first plot in Figure 4 shows the direct comparison between the measured speed of the tip of the beam resulting from the execution of the motion profile generated by the ZV shaper and by the proposed variational method under 'nominal' conditions, i.e. without any explicit account for robustness. The tip speed is estimated by integrating the signal generated by the accelerometer mounted on the tip of the beam. Motion happens between $t = 1$ s and $t = 2$ s, and therefore the motion of the tip of the beam happening after $t = 2$ s can be regarded as the effect of a residual vibration.

While the two method appear to produce residual vibrations with almost identical amplitudes, it is clearly visible that the variational method leads to smaller elastic displacements during motion. A similar result is observed when

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looking at the second plot of Fig. 4, which shows the results of the application of the motion profiles generated in one case by the ZVD shaper and the one generated by the robust variational solution: in this case the latter lead to less pronounced oscillations both during motion and after motion completion. The presence of residual vibration, which should not happen under nominal conditions in any of the mentioned test, are mainly due to the limited accuracy of the robot when reproducing the planned motion profile. The analysis of

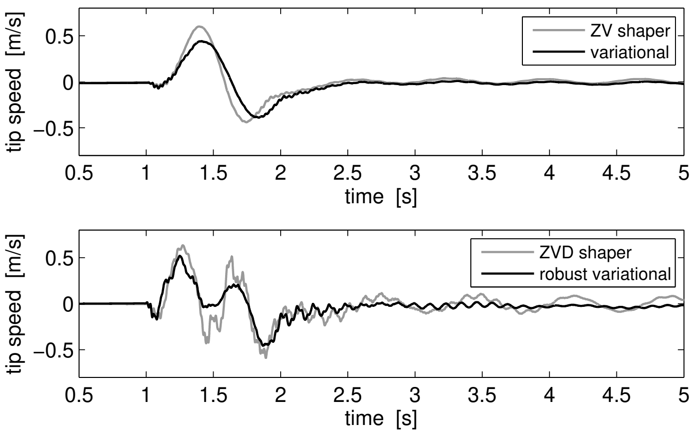

Figure 4: Tip speed: nominal and robust variational solutions vs. ZV and ZVD shaper

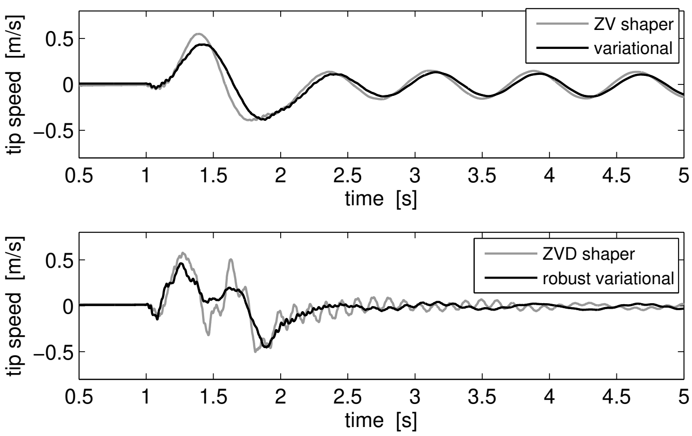

Figure 5: Tip speed: nominal and robust variational solutions vs. ZV and ZVD shaper, perturbed system

the performance of resulting from the application of the proposed planning methods is extended to include an evaluation of their robustness properties. The same motion profiles have therefore been tested on the perturbed system. The modification has the effect of lowering the beam oscillating frequencies. The

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results are presented in Fig. 5: the first plot highlights the slightly improved performance resulting from the application of the nominal variational method, which shows less pronounced vibration both during the motion and after its completion. However, both method can be judged as scarcely robust, being unable to properly react to a system perturbation.

A significant robustness improvement, which is testified by less pronounced residual vibrations, is brought by both 'robust' approaches, with a slightly better behavior sported by the proposed robust variational method, according to the second plot in Fig. 5. The latter, when compared to the ZVD shaper is, again, capable of a less pronounced excitation of the vibration phenomena both during motion and after motion completion.

5 Conclusion

This paper has presented the preliminar experimental validation of a model-based motion planning method applied to flexible mechanisms. The method is based on the definition and solution of a variational problem, which can include robustness specifications too. The dynamic model used for the motion planning can be augmented with the inclusion of its sensitivity functions with respect to an uncertain parameter. The experimental data presented in this work provide an indication of the effectiveness of this method. The effectiveness of the proposed method is also shown by a direct comparison with the ZV and ZVD input-shaping method, which represent the de-facto standards for the suppression of motion-induced residual vibrations.

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