Does inertia matching imply energy efficiency?

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Abstract

The aim of this work is to analyze the problem of designing energy-efficient servo driven systems, by providing a choice of the proper motor size and the correct gear ratio for an arbitrary load. The analysis takes into account a typical test case and a large number of possible design, which are analyzed by parameterizing the design space on the basis of the motor size and reduction ratio, and on the inertia ratio as well. The analysis of the feasibility of each design, and of the resulting energy consumption for each design sample, show that the most common sizing guideline, based on inertia matching, is incapable of capturing the most energy efficient design.

Keywords: SDG12; Energy efficiency; Servo system design; Brushless motor; Gearbox; Inertia matching

1 Introduction

Servo-controlled systems are the core elements of most mechatronic systems, automatic machines and robots, and as such are widely adopted in industry. Their relevance in countless applications is also testified by their impact on the overall energy consumption by the industry: several studies [4, 6] indicate that electric motors are responsible for up to 70% of such energy consumption. The relevance of servo-driven system, and the push by government policies is now fostering the development and the adoption of 'greener' solutions also in mechatronic systems, which can help meeting the goal of more sustainable consumption and production set by the Sustainable Development Goal 12 [5]. As a result, many manufacturers now offer solutions characterized by high efficiency [12, 7], which in the case of servo-drive systems means improving the efficiency of the motor, of its electronic driver, and of the gearbox, being these its key components.

Choosing 'premium' components is not the only - or possibly the best - path to a greener industry: several studies have shown that the available options for the improvement of energy efficiency are several, as different approaches can lead to non-negligible improvements [3]. A consolidated design principle supporting energy efficiency is the adoption of lightweight components or of

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springs to store and release, at proper time, some mechanical energy [8, 11, 2], using an approach usually referred to as 'natural motion' [13]. Alternatively, the energy consumption can be minimized by a careful motion design, since a careful tuning of the acceleration profile can be beneficial in the reduction of the inertial forces to be counteracted by the actuators, which are usually the most relevant contribution to the overall energy consumption [1, 11].

Energy-efficiency can be, however, tackled starting from the initial stages of the design of a machine: in particular by carefully choosing the electric drive, the motor and the gearbox (if any), a naturally more efficient solution can be implemented. Despite the relevance of the problem, it is recognized that choosing the wrong size motor for the application is one of the most common mistake in system design [12].

The most used approach to the design of a mechatronic systems, which essentially boils down to the proper choice of a motor and a reduction gear from some catalogs, follows a procedure in which the reducer is chosen first. This choice comes first since the design is usually driven by an estimation of the load parameters - i.e. load speed and torque profiles are usually known in good detail. The choice of the motor comes after, and consists in a process which discards the unsuitable ones: among the non-discarded solutions, the cheaper options is usually referred to as the 'best one'. Such approach has several shortcomings: first of all, the procedure might require some iterations, as the initial choice of the gearbox might be non-compatible, or good enough, when considered together with the population of available motors. Moreover, in this procedure the energy consumption is generally not taken into consideration, and in most cases it does not lead to an energy efficient solution. Manuals often suggest a design procedure based on simple analytic relationships, the most common one being based on the achievement of inertia matching [9]. This work will briefly analyze the problem of designing energy efficient servo-driven systems, and the generally detrimental effect of inertia matching on energy consumption. The data gathered from datasheet will be used to test hundreds of different design, which are checked for feasibility and energy requirement.

2 System modeling

A typical servo-driven system can be represented by the diagram shown in fig.1(a), as a cascade of a motor, a gearbox, and a load. Assuming that the load motion profile is known, it can be represented as a time-varying resistant torque profile $T_2(t)$; hence, assuming that both Coulomb and viscous friction act on the motor shaft, the motor should provide the following torque:

$$T_m(t) = (J_m + J_r)\,\ddot{\vartheta}_l(t) i + k_v \dot{\vartheta}_l(t) i + T_f + \frac{T_2(t)}{i} \tag{1}$$

being J_m and J_r the motor and reduced moment of inertias, k_v the viscous friction constant, T_f the Coulomb friction torque, and i the gear ratio. Assuming an equivalent DC-motor description of the brushless motor that drives the

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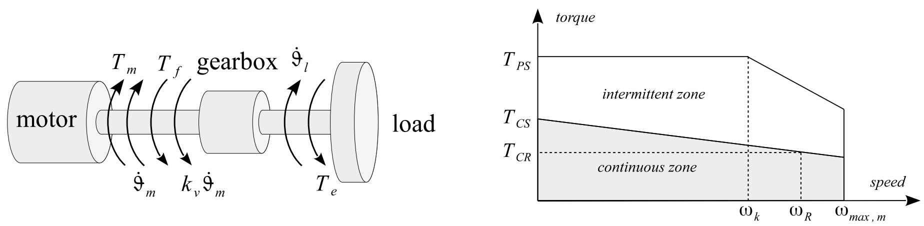
system, the voltage drop across the motor leads and the current absorption can be estimated as:

$$V(t) = RI(t) + k_b \vartheta_m(t); \qquad I(t) = \frac{T_m(t)}{k_t}; \tag{2}$$



(a) (b)

Figure 1: System modeling (a) and motor characteristic curve (b)

in which k_t is the motor torque constant and k_b is the back-emf constant. This simple, but generally accurate model allows to predict the motor electric power absorption as a voltage to current product, which, being then integrated over time, provides the overall energy consumption over a time frame $[0, T]$ as:

$$E_m = \int_0^T VIdt = \int_0^T RI^2dt + \int_0^T k_b \dot{\vartheta}_m Idt = \underbrace{\frac{R}{k_t^2}\int_0^T T_m^2 dt}_{\text{Joule losses}} + \underbrace{\frac{k_b}{k_t}\int_0^T \dot{\vartheta}_m T_m dt}_{\text{mechanical losses}} \tag{3}$$

Equation (3), together with (1) provide an energy estimation model that can be used to forecast the energy consumption of a given design, which is directly usable if the proper motor and gearbox characteristics can be drawn from catalog data. Each design must however be checked for feasibility, i.e. it must be checked that the maximum and average rating listed in datasheets are not exceeded. In order to ensure feasibility, the gear ratio must be checked against the resulting maximum gearbox speed, ω_{1B}, against the gearbox rated speed, ω_{1r}, and against maximum motor speed, $\omega_{m,max}$. The last data is also represented in the motor speed-torque characteristics as shown in fig. 1(b). The gearbox torque capability suggested by the manufacturer should not be exceeded as well, hence the impact of the peak load toque, T_2^{max}, and of the root-mean-cubic value of the load torque, T_2, should be checked as well. For a detailed treatment of such design boundaries please refer to [10]. The motor size is to be checked as well, as to ensure that no overheating conditions are encountered during normal operation: to do so, the motor torque-speed pairs should be checked to ensure that the motor RMS operative point lies in the continuous operative zone, which is shown in fig.1(b). Without going into details, for which, again, the reader should refer to [10], a correct operation of the motor can be

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ensured by a sufficiently large motor size, i.e. by ensuring that its continuous torque at stall, T_{CS}, is large enough, ad for an adequately rated gearbox. Within this limitations, and according to the energy requirements set by equations (1,3), the relationship between the two main parameters of the design, i and T_{CS}, and the energy consumption can be investigated. An analysis of this kind is provided in the next section, by choosing a benchmark load and by testing a large number of design using catalogs data. A list of 45 motors and of 17 reducers, which leads to 765 possible design, is tested and feasibility, together with energy efficiency, is verified for each possible choice.

2.1 Inertia matching conditions

The concept of inertia matching is well established in engineering practice, and has been, for a long time, suggested in several application books and manufacturers' datasheet. The Inertia Ratio (IR) is defined as the ratio between the reflected load moment of inertia (plus the reducer inertia) and the motor inertia: as shown in the work [9], in such condition the motor effort, for a given load acceleration, is minimized. IR has has gained a noticeable popularity as it has been shown to minimize the risk of instability, which can occur if the control tuning cannot compensate the effects of backlash and elasticity, which are mostly due to the non-ideal characteristics of the gearbox. The development of modern control systems, and the general increase of the bandwidth of commercially available servo control systems, has however limited the impact of such problem, and rarely closed-loop instability is caused by an inertia mismatch. In order to pursuit inertia matching, the transmission ratio is set as $i_{IM} = \sqrt{J_m/J_l}$. This choice simplifies the design procedure, allowing to choose first the motor and then the gear ratio. The problem is however, and in general, more complex, as both the motor choice and the gear ratio i do affect the performance of the servo-drive system to be designed. Hence, both the choice of the gearbox and of the motor should be made within a coherent and possibly concurrent design approach. If energy optimality is sought, the traditional design approaches should be revisited, in particular, as it will be shown with a simple test-case, aiming for minimum (or close to minimum) motor sizing or to unitary Inertia Ratio is not, in most cases, the best energy-wise option. Usually a minimum size motor is chosen to reduce the impact of its inertia on the overall motor torque, however the effects of a larger motor inertia can be compensated by an improved torque generation, i.e. by a higher motor torque constant.

3 Design analysis

The test-case under consideration is built by considering a load with inertia $J_l = 2\ kgm^2$, on which act a constant resistant torque $T_e = 0.5\ Nm$ and a viscous friction torque, represented by the load-side viscous friction constant $k_v = 1 \times 10^{-2}\ Nms/rad$. The load is moved according to a trapezoidal motion profile, with $\lambda = .2$, over a duration $T = 3\ s$. Each motion sequence, which

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provides a load displacement equal to $h = 15\pi\ rad$, is followed by a pause interval whose duration is $T_p = 1\ s$. The resulting load-side speed and torque pairs are shown in fig. 2.

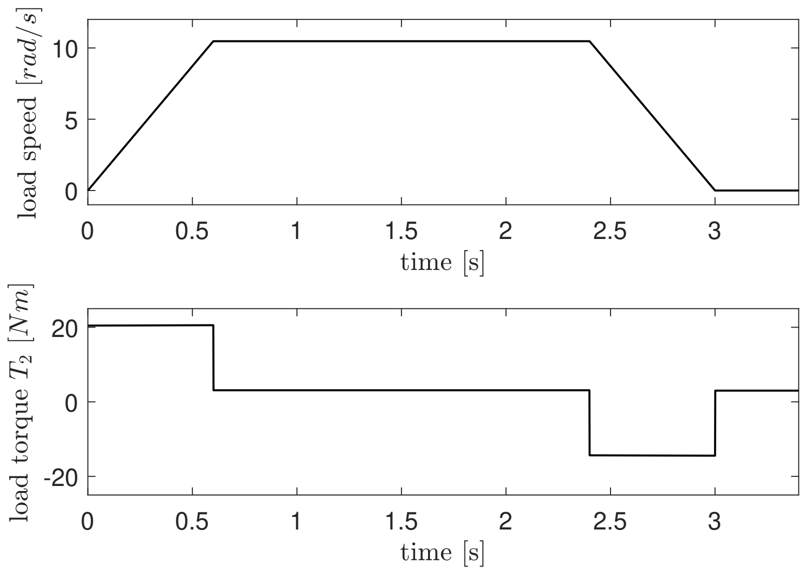

Figure 2: Load speed ω_l and load torque T_2

The aforementioned 765 design solutions are then verified for feasibility, and the respective energy requirements are represented in fig. 3, which plots the overall energy consumption, E_m, as a function of i and of the motor size T_{CS}. The best energy-wise design is highlighted by the black circle.

The motors on right-side of the figure are the ones with the smallest continuous stall torque, hence the 'smallest' ones, i.e. the energy-optimal solution is found for an oversized motor, and for a relatively small reduction ratio. The main features of the energy-optimal design are shown in table 1, which also highlights that the energy improvement over the minimum size design is rather relevant, being equal to 25%. The latter design requires a motor with $T_{CS} = 0.84Nm$ and a $40:1$ reduction: the resulting Inertia Ratios are, respectively, equal to 12.11 and 41.76. Hence both design deviate radically form the inertia matching condition.

Plotting the total energy consumption for the 501 feasible designs among the 765 possible ones versus the inertia ratios results in the logarithmic plot in figure 4: the analysis suggests that the inertia ratio should be chosen between 5 and 25. The inertia matching condition does not appear to be energetically sound, as a unitary Inertia Ratio desing requires, in the best case, an energy equal to 111.4 J: this figure is almost twice the minimum energy figure.

This analysis, whose results are also corroborated by several other test-cases

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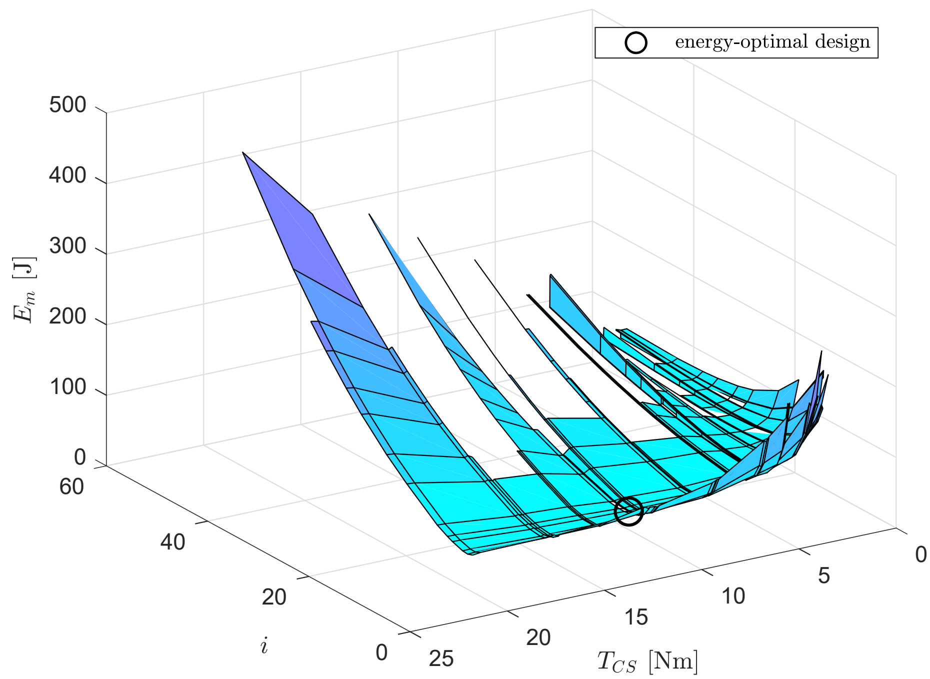

Figure 3: Energy consumption vs. T_{CS} and i: the energy-optimal design is shown by the black circle

Table 1: Comparison of motor-gearbox designs

Design objective	gear ratio i	T_{cs} $[Nm]$	IR	Energy consumption $[J]$	% variation
Minimum energy	7	11.93	12.11	66.1	-
Unitary IR	25	11.93	0.98	111.4	+68.5%
Minimum motor size	40	0.84	41.86	88.8	+34.3%

not reported here, suggest that an energy-efficient design is generally found for oversized motors and for inertia ratios larger than 5, as designs with unitary IR and close to minimum size motors are rather inefficient. The rather generous oversizing suggested by the energetic analysis can be explained by the results provided in fig. 5, which shows how the two most-relevant motor parameters are scaled with the motor size. The ratio R/k_t^2, whose reciprocal is the square value of the motor constant, falls rapidly for any increase of T_{CS}: hence the energy associated by the Joule losses, as highlighted in eq.(3), can be reduced by choosing larger motors. Certainly a larger motor implies a larger inertial load, which affects both the magnitude of both Joule and mechanical losses, however this effect has a limited impact, since it is usually overcompensated by the change in the ratio R/k_t^2. The analysis of the nameplate data of the 45 motors under consideration shows that the proportionality between J_m and

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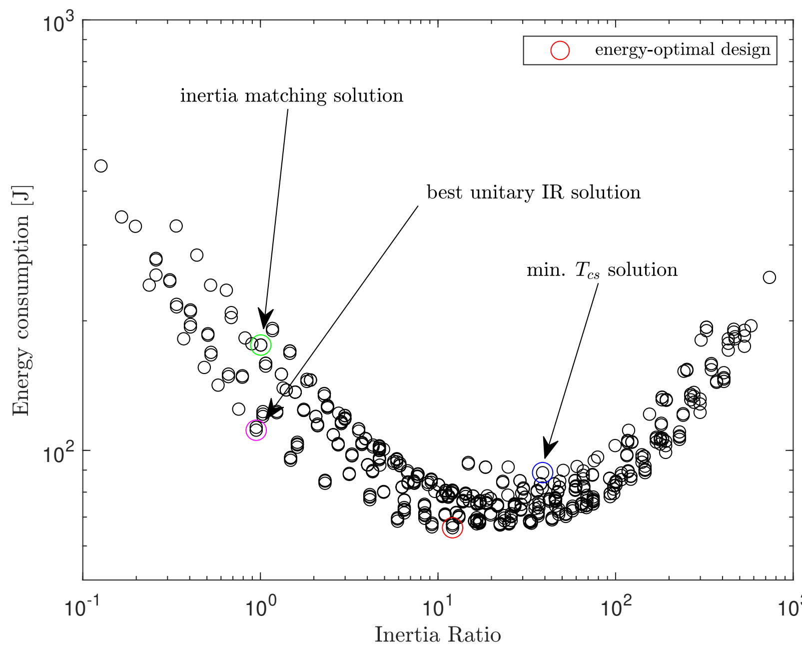

Figure 4: Energy consumption vs. inertia ratio for test case II: the energy-optimal design is shown by the red circle

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T_{CS} is almost linear, as it can be seen in fig.5. Hence, the effects of the decrease of R/k_t^2 with the increase of T_{CS} prevails over the effects of a larger T_m, and as a results a larger (but not too large) motor can be energetically more efficient.

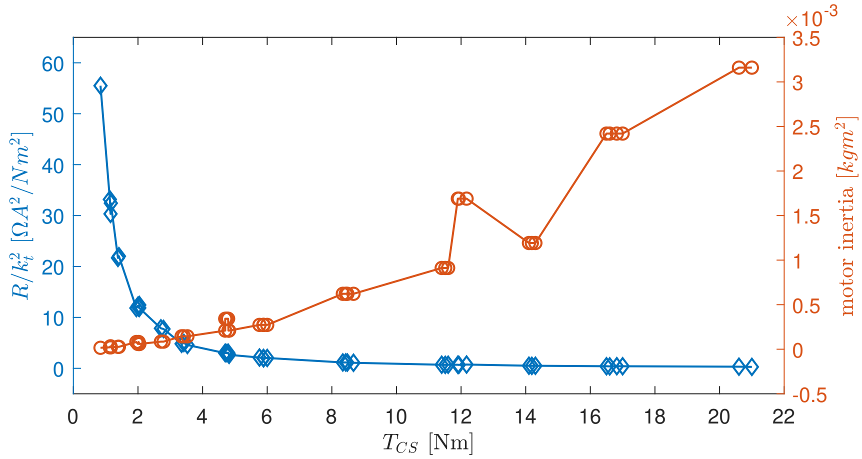

Figure 5: R/k_t^2 ratio vs. T_{CS} for the 45 motor samples

4 Conclusion

In this work an analysis on the impact of a classic design rule, i.e. the inertia matching condition, is re-considered from the energy consumption point of view. Starting form an estimation model, which is used to forecast the energy consumption of a servo-driven system, a large number of possible designs are tested. The results highlight the complex relationship between design parameters and energy consumption, which calls for an integrated motor-gearbox design procedure. Moreover, it is found that targeting for inertia matching and minimum motor size are both, energy-wise, inefficient choices. In general, is it found that efficiency can be boosted by using large inertia ratios and oversized motors.

References

[1] Paolo Boscariol, Roberto Caracciolo, Dario Richiedei, and Alberto Trevisani. Energy optimization of functionally redundant robots through motion design. *Applied Sciences*, 10(9):3022, 2020.

[2] Giovanni Carabin, Ilaria Palomba, Erich Wehrle, and Renato Vidoni. Energy expenditure minimization for a delta-2 robot through a mixed approach. In *European Congress on Computational Methods in Applied Sciences and Engineering*, pages 383–390. Springer, 2019.

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https://link.springer.com/chapter/10.1007/978-3-030-87383-7_31

[3] Giovanni Carabin, Erich Wehrle, and Renato Vidoni. A review on energy-saving optimization methods for robotic and automatic systems. *Robotics*, 6(4):39, 2017.

[4] A De Almeida, J Fong, CU Brunner, R Werle, and M Van Werkhoven. New technology trends and policy needs in energy efficient motor systems - a major opportunity for energy and carbon savings. *Renewable and Sustainable Energy Reviews*, 115:109384, 2019.

[5] Sandra Guevara and Isabel Pla Julián. Sustainable consumption and production: A crucial goal for sustainable development - reflections on the spanish sdg implementation report. *Journal of Sustainability Research*, 1(2), 2019.

[6] Shyi-Min Lu. A review of high-efficiency motors: Specification, policy, and technology. *Renewable and Sustainable Energy Reviews*, 59:1–12, 2016.

[7] Marek Lukaszczyk. Improving efficiency in electric motors. *World Pumps*, 4:34–41, 2014.

[8] Ilaria Palomba, Erich Wehrle, Giovanni Carabin, and Renato Vidoni. Minimization of the energy consumption in industrial robots through regenerative drives and optimally designed compliant elements. *Applied Sciences*, 10(21):7475, 2020.

[9] K.A. Pasch and W.P. Seering. On the drive systems for high-performance machines. *Journal of Mechanisms, Transmissions, and Automation in Design*, 106(1):102–108, 1984.

[10] Dario Richiedei. Integrated selection of gearbox, gear ratio, and motor through scaling rules. *Mechanics based design of Structures and Machines*, 46(6):712–729, 2018.

[11] Dario Richiedei and Alberto Trevisani. Analytical computation of the energy-efficient optimal planning in rest-to-rest motion of constant inertia systems. *Mechatronics*, 39:147–159, 2016.

[12] Rahman Saidur. A review on electrical motors energy use and energy savings. *Renewable and Sustainable Energy Reviews*, 14(3):877–898, 2010.

[13] Lorenzo Scalera, Giovanni Carabin, Renato Vidoni, and Theeraphong Wongratanaphisan. Energy efficiency in a 4-dof parallel robot featuring compliant elements. *Int. J. Mech. Control*, 20(02), 2019.

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